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The Effect of Random Vibration Coefficients on Differential Equations

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Abstract

We study in this paper the value of the first moment in the stochastic differential equation, when the equation contains influential forces and these forces are colored noise. This equation describes the state of the first moment in the random movement of things. Also, we study many properties of dynamic in the case of the first dimension. This paper also contains some graphics and tables of moment and the results of the extracted equation.

Keywords: First moment, Colored Noise, White noise, the work internal energy.

تأثير معاملات الاهتزاز العشوائي على المعادلات التفاضلية

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قسم الرياضيات ،كلية التربية للعلوم الصرفة ،جامعة ذي قار ،العراق

الخلاصة

سوف ندرس في هذا البحث قيمة العزم الاول في المعادلة العشوائية التفاضلية المقترحة عندما تحتوي هذه المعادلة على قوة مؤثرة هي قوة الضوضاء كما يوضح البحث حالة العزم الاول للنظام في الحركة العشوائية للاشياء. كما سندرس بعض الخواص الديناميكية في حالة النظام ذات البعد الواحد. كما قمنا برسم العزم الاول بعد استخراج النتائج.قمنا بتكوين جداول لبعض القيم المقترحة للمعاملات العشوائية

1. Introduction

In1980, there are many studies that have addressed the colored noise under the influence of vibration. The stochastic equation is used in several sciences as physical, mechanical, energy and velocity. The stochastic dynamic oscillator [1] is study the effects of colored noise on the stochastic equation, and this noise intensively studied in many published researches, and study the values of records and their associated statistic are of great importance in multiply real life situations such as meteorology, seismology, sporting events and life test. Also, in this paper we drive several steps to form the first moment [2], and one of phenomena to study are some dynamic properties in one dimension [3,4], explanation of entropy behavior by probability density function [5], these systems of equation are produced through many complex factors, but the reason for studying them is that they are statistical, but it contain many doubts and errors, Also, scientists have enhanced the stability of the noise see [9]. As we note in stochastic differential equations, they contain a Brownian motion which is stochastic process when it has some properties see [8]. Finally in this paper we study two sections, one it has many steps to solve the first moment [2]. And in two section we find many properties of dynamic after several derivations [3,6,11] when we take one system and this system contains an oscillator with random frequency.

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And our system that we use in the solution is defined as following:

$$\frac{dv}{dt} = \frac{-2a}{r}v - \frac{b^2}{r}(1+z)q + \frac{\alpha}{r}w.$$
$$\frac{dq}{dt} = v.$$

q *is position,* v *is velosity for the system* , r mass ,
z *is colored noise* , b^2 *is a frequancy constant and* a *is a friction*

2. First Moment

In this section, we proposed a specific system to find the result of the first moment when we use many equations and some derivative. We also used MATLAB to solve the final results.

Our system is:

$$\frac{dq}{dt} = v \tag{1}$$

$$\frac{d}{dt}v = \frac{-2a}{r}v - \frac{b^2}{r}(1+z)q + \frac{\alpha}{r}w. \tag{2}$$

When we multiply (1) by z we get $\frac{dq}{dt}z = zv$

By Shapiro –Loginov [10] and average we get

$$\frac{d}{dt}\langle zq\rangle = \langle zv\rangle - \lambda\langle zq\rangle \tag{3}$$

Takes average to eq. (2) it is becomes as

Remark (we mean by this symbol $\langle..\rangle$ is a mathematical system)

$$\frac{d}{dt}\langle v\rangle = \frac{-2a}{r}\langle v\rangle - \frac{b^2}{r}(1+z\langle q\rangle + \frac{\alpha}{r}\langle w\rangle. \tag{4}$$

Multiply (4) by z we have

$$\frac{d}{dt}\langle vz\rangle = \frac{-2a}{r}\langle vz\rangle - \frac{b^2}{r}(z+z^2)\langle q\rangle + \frac{\alpha}{r}\langle zw\rangle. \tag{5}$$

When z is independent of w [1] we have :

$\langle zw \rangle$ =0 with $z^2 = \delta^2$

Substitute these values in (5) we get

$$\frac{d}{dt}\langle vz \rangle = \frac{-2a}{r}\langle vz \rangle - \frac{b^2}{r}(z+\delta^2)\langle q \rangle. \quad (6)$$

Now multiply the equation (1) by ($\frac{d}{dt}+\frac{2a}{r}$) then we get that :

$$(\frac{d^2}{dt^2}+\frac{2a}{r})\langle q \rangle = (\frac{d}{dt}+\frac{2a}{r})\langle v \rangle \quad (7)$$

From equation (2) we get:

$$(\frac{d}{dt}+\frac{2a}{r}) < v > = \frac{-b^2}{r} < q > - \frac{b^2}{r} < zq > \quad (8)$$

From equation (7) we have:

$$(\frac{d^2}{dt^2}+\frac{2a}{r})\langle q \rangle = \frac{-b^2}{r} < q > - \frac{b^2}{r} < zq > \quad (9)$$

Now convey the value $\frac{-b^2}{r} < q >$ to the left side of the equation (9) becomes as:

$$(\frac{d^2}{dt^2}+\frac{2a}{r}+\frac{b^2}{r})\langle q \rangle = -\frac{b^2}{r} < zq > \quad (10)$$

Divide equation (9) on $-\frac{b^2}{r}$ then it becomes :

$$< zq > = -\frac{r}{b^2} \ (\frac{d^2}{dt^2}+\frac{2a}{r}+\frac{b^2}{r})\langle q \rangle \quad (11)$$

From equation (3) we have that:

$$(\frac{d}{dt}+\lambda)\langle zq \rangle = \langle zv \rangle \quad (12)$$

From equations (11) and (12) then:

$$(\frac{d}{dt}+\lambda) \ (-\frac{r}{b^2} \ (\frac{d^2}{dt^2}+\frac{2a}{r}+\frac{b^2}{r})\langle q \rangle) = \langle zv \rangle \quad (13)$$

When we distribute the amount on the bow then, we have:

$$(\frac{-r}{b^2}\frac{d^3}{dt^3} - (\frac{2a+b^2}{b^2})\frac{d}{dt} - \frac{\lambda r}{b^2}\frac{d^2}{dt^2} - (\frac{2a+b^2}{b^2})\lambda)\langle q \rangle = \langle zv \rangle \quad (14)$$

From equation (6) we have:

$$(\frac{d}{dt}+\frac{2a}{r}) < zv >= -\frac{b^2}{r}\langle zq\rangle + (-\frac{b^2}{r}\delta^2)\langle q\rangle \quad (15)$$

From equation (11) then:

$$(\frac{d}{dt}+\frac{2a}{r}) < zv >= -\frac{b^2}{r}(-\frac{r}{b^2}(\frac{d^2}{dt^2}+\frac{2a}{r}+\frac{b^2}{r})\langle q\rangle) \quad (16)$$

After many steps, we get the following equation:

$$\frac{d^4}{dt^4}\langle q\rangle = (\frac{-(\lambda r+2a)}{r})\frac{d^3}{dt3}\langle q\rangle - (\frac{2a+2a\lambda+2b^2}{r})\frac{d^2}{dt^2}\langle q\rangle - (\frac{2a+b^2}{r}\lambda+\frac{4a^2+2ab^2}{r})\frac{d}{dt}\langle q\rangle - (\frac{4a^2+2ab^2}{r^2}\gamma+\frac{2a+b^2-\delta^2 b^4}{r^2})\langle q\rangle \quad (17)$$

Let us suppose the values:

$$G=\frac{(\lambda r+2a)}{r} \qquad H=\frac{2a+2a\lambda+2b^2}{r} \qquad N=\frac{2a+b^2}{r}\lambda+\frac{4a^2+2ab^2}{r}$$

$$P=\frac{4a^2+2ab^2}{r^2}\lambda+\frac{2a+b^2-\delta^2 b^4}{r^2}$$

To substitute in the equation (17), then:

$$\frac{d^4}{dt^4}\langle q\rangle = [-G\frac{d^3}{dt^3}-(H)\frac{d^2}{dt^2}-N\frac{d}{dt}-(P)]\langle q\rangle \quad (18)$$

Suppose that

$$q_0 = c_0 \qquad q_0{}' = c_1 \qquad q_0{}'' = c_2 \qquad q_0{}''' = c_3$$

Use the MATLAP to solve differential equations from the fourth rank we suppose many values as:

$c_0 = 0\, and \quad c_1 = 0.5 \quad c_2 = 1 \quad c_3 = 1 \quad and \quad where \quad r = 1$

$a = 1 \quad \delta^2 = 1 \quad b^2 = 1 \quad \gamma = 0.5 \quad then the \quad root \quad as \quad B_1 = 0.3632 + 2.12141$

$B_2 = -0.3632 - 2.1214i$

$B_3 = -0.8868 + 0.5412i$

$B_4 = -0.8868 - 0.5412i$

The general solution of the first moment to ( ) becomes as:

$$< q >= c_0 e^{B_1 t} + c_1 e^{B_2 t} + c_2 e^{B_3 t} + c_3 e^{B_4 t}$$

When we make up the value in above equation of general solution we get:

$$< q >= 0.5e^{(-0.3632-2.1214i)t} + e^{(-0.8868+0.5412i)t} + e^{(-0.8868-0.5412i)t}$$

now if the value as

$r = 0.3 \; a = 1.5 \quad \gamma = 1 \quad \delta^2 = 0.5 \quad b^2 = 0.2$ *then the root become* $B_1 = -3.2616 + 2.4870i$

$B_2 = -3.2616 - 2.4870i$

$B_3 = 1.0949 + 2.6196i$

$B_4 = 1.0949 - 2.6196i$ *from this value the general soluation become as*

$$< q >= 0.5e^{(-3.2615-2.4870i)t} + e^{(1.0949+2.6196i)t} + e^{(1.0949+2.6196i)t}$$

if $r = 1.3 \quad a = 1.5 \quad \gamma = 1.5 \quad \delta^2 = 1.1 \quad b^2 = 1.3$

then the roots become as

$B_1 = -1.3705 + 1.9753i$

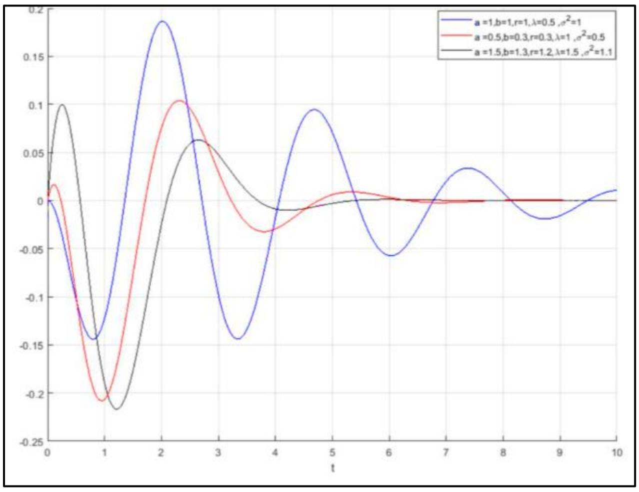
$B_2 = -1.3705 - 1.9753i$

$B_3 = -0.6295 + 1.3926i$

$B_4 = -0.6295 - 1.3926i$

the general solution become as

$$< q >= 0.5e^{(-1.3705-1.9753i)t} + e^{(-0.6295+1.3926i)t} + e^{(0.6295-1.3926i)t} \qquad .(19)$$



Figuer-1 Drawing the first moment of the differential equation under
$\lambda = 1 \quad \sigma^2 = 1 \quad a = 1 \quad b = 1 \quad r = 1 \quad a = 0.5 \quad b = 0.3 \quad r = 0.3 \quad \lambda = 0.5$

We take assume some values for (t) by creating tables containing a set of approximate values

Table-1 In this table We will choose some parameter values as follows

a=0.5 r=0.3 $b^2 = 0.2$ $\lambda = 1$ $\alpha = 0.5$

t	$< q^2 >$	t	$< q^2 >$	t	$< q^2 >$	t	$< q^2 >$	t	$< q^2 >$
0	0	1.0101	1.33365	2.0202	2.73803	3.0303	3.693	4.0404	4.32521
0.05051	0.00804	1.06061	1.41476	2.07071	2.79557	3.08081	3.73121	4.09091	4.35046
0.10101	0.03046	1.11111	1.49496	2.12121	2.85196	3.13131	3.76864	4.14141	4.37519
0.15152	0.06497	1.16162	1.57418	2.17172	2.90723	3.18182	3.8053	4.19192	4.39941
0.20202	0.1096	1.21212	1.65233	2.22222	2.96139	3.23232	3.84121	4.24242	4.42314
0.25253	0.16264	1.26263	1.72937	2.27273	3.01447	3.28283	3.87638	4.29293	4.44638
0.30303	0.22266	1.31313	1.80525	2.32323	3.06648	3.33333	3.91083	4.34343	4.46914
0.40404	0.35879	1.41414	1.95339	2.42424	3.16738	3.43434	3.97764	4.44444	4.51327
0.45455	0.4329	1.46465	2.02562	2.47475	3.2163	3.48485	4.01001	4.49495	4.53466
0.50505	0.50997	1.51515	2.0966	2.52525	3.26423	3.53535	4.04173	4.54545	4.55561
0.60606	0.67032	1.61616	2.23478	2.62626	3.35719	3.63636	4.10322	4.64646	4.59624
0.65657	0.75255	1.66667	2.30199	2.67677	3.40226	3.68687	4.13302	4.69697	4.61592
0.70707	0.83557	1.71717	2.36795	2.72727	3.4464	3.73737	4.16221	4.74747	4.63521
0.75758	0.91902	1.76768	2.43267	2.77778	3.48965	3.78788	4.19081	4.79798	4.65409
0.80808	1.0026	1.81818	2.49615	2.82828	3.53202	3.83838	4.21881	4.84848	4.67259
0.85859	1.08605	1.86869	2.55841	2.87879	3.57352	3.88889	4.24624	4.89899	4.69071
0.90909	1.16915	1.91919	2.61947	2.92929	3.61417	3.93939	4.27311	4.94949	4.70846

Table-2 In this table We will choose some parameter values as follows:

$a = 1.5$ $b^2 = 1.3$ $\lambda = 1.5$ $\alpha = 1.1$ $r = 1.2$

t	$< q^2 >$	t	$< q^2 >$	t	$< q^2 >$	t	$< q^2 >$	t	q^2
0	0	1.0101	0.86178	2.0202	0.96519	3.0303	0.86986	4.0404	0.88588
0.05051	0.00611	1.06061	0.89158	2.07071	0.9571	3.08081	0.86953	4.09091	0.88661
0.10101	0.02338	1.11111	0.9178	2.12121	0.94904	3.13131	0.86949	4.14141	0.88726
0.20202	0.08523	1.21212	0.95979	2.22222	0.93339	3.23232	0.87012	4.24242	0.88836
0.25253	0.12687	1.26263	0.97582	2.27273	0.92597	3.28283	0.87073	4.29293	0.88879
0.30303	0.17383	1.31313	0.98876	2.32323	0.91891	3.33333	0.87149	4.34343	0.88916
0.35354	0.22485	1.36364	0.9988	2.37374	0.91227	3.38384	0.87238	4.39394	0.88947
0.40404	0.27875	1.41414	1.00615	2.42424	0.90608	3.43434	0.87337	4.44444	0.88971
0.60606	0.50335	1.61616	1.01313	2.62626	0.88633	3.63636	0.87784	4.64646	0.89015
0.65657	0.55768	1.66667	1.01042	2.67677	0.88267	3.68687	0.87899	4.69697	0.89015
0.70707	0.61003	1.71717	1.00637	2.72727	0.87951	3.73737	0.88012	4.74747	0.89012
0.75758	0.6599	1.76768	1.0012	2.77778	0.87684	3.78788	0.88122	4.79798	0.89005
0.80808	0.70693	1.81818	0.99511	2.82828	0.87463	3.83838	0.88227	4.84848	0.88996
0.85859	0.75081	1.86869	0.98829	2.87879	0.87285	3.88889	0.88326	4.89899	0.88985
0.90909	0.79134	1.91919	0.98092	2.92929	0.87149	3.93939	0.8842	4.94949	0.88972
0.9596	0.82835	1.9697	0.97317	2.9798	0.8705	3.9899	0.88507	5	0.88958

The above tables show how the values of the first moment change when we take a different value to the constants at each time and we notice that this moment changes in every assumed value.

3. Many Properties of Thermodynamic in one-dimensional

In this part, we will discuss some dynamic properties, through a proposed one-dimensional system. To extract these properties, we derivations and integrals to get the solution.

Our model in one-dimensional as:

$$dq = \frac{-b^2}{2a}(1+\alpha z)qdt + \frac{\propto \sqrt{2\pi D}}{2a} dBt \qquad \alpha = 1 \tag{20}$$

When we based on [3] then we have an internal energy U(q), it's write as

$$U(q,t) = \frac{-b^2}{4a}(1+z)q^2 \tag{21}$$

Then also we need the next value:

$$U(q_0) = -\frac{b^2}{4a}(q^2)$$

The partial of internal energy is

$$\frac{\partial U}{\partial t} = -\frac{b^2}{4a}(zq^2)$$

From [3] the law of work becomes as:

$$W = \int_0^t \frac{\partial U(q,t')}{\partial t'} dt' \tag{22}$$

$$W = \frac{-b^2}{4a}\int_0^t z'_s\, q_s ds \tag{23}$$

When we take the expectation of work then we have:

$$\langle W \rangle = \frac{-b^2}{4a}\int_0^t \langle z'_s\, q_s \rangle ds \tag{24}$$

Also we extract the dissipated heat (Q) by using the following law [3]:

$$Q = W - \Delta U \tag{25}$$

In order for us to find the value of the dissipated heat in the above law, we must be able to find a value of the change internal energy $-\Delta U$ by using:

$$\Delta U = U(q,t) - U(q_0) \tag{26}$$

$$U(q_0) = \frac{-b^2}{4a} q^2 \tag{27}$$

After a few simplifications, we get that the change of internal energy as follows:

$$\Delta U = \frac{-b^2}{4a} z q^2 \tag{28}$$

Now we can apply the relationship law (25) and it becomes:

$$Q = \frac{-b^2}{4a} \int_0^t z'_s q_s \, ds + \frac{-b^2}{4a} z_t q^2$$

$$Q = \frac{-b^2}{4a} \left(- \int_0^t z'_s q_s \, ds - z_t q^2\right) \tag{29}$$

Through the above results obtained, we can extract a new result, which is the change in the environment entropy produced by the heat in the time interval using the follow relation:

$$\Delta S_m = \frac{Q}{T} \tag{30}$$

$$\Delta S_m = \frac{1}{T}\left(\frac{-b^2}{4a}\left(- \int_0^t z'_s q_s \, ds - z q^2\right)\right. \tag{31}$$

When T is the temperature.

Now find entropy [3] by using following steps:

$$S(t) = -\int P(q,t) \, LnP(q,t) \quad dq \tag{32}$$

P (q ,t) is probability density function pdf when we based on [6,7,11] we can write the probability density function as:

$$P(q,t) = \frac{1}{\sqrt{\frac{2\pi(\alpha^2 \pi D)}{b^2(1+\alpha D)}}} \exp \frac{-\left[q - q_0 e^{\frac{-b^2}{2}(1+\alpha D)t}\right]^2}{2\frac{\alpha^2 \pi D}{b^2(1+\alpha D)}} \tag{33}$$

Then the entropy is become as:

$$S(t) = -\int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{\frac{2\pi(\alpha^2\pi D)}{b^2(1+\alpha D)}}} \exp \frac{-\left[q - q_0 e^{\frac{-b^2}{2}(1+\alpha D)t} \right]^2}{2\frac{\alpha^2 \pi D}{b^2(1+\alpha D)}} \left(-Ln\sqrt{\frac{2\pi(\propto^2 \pi D)}{b^2(1+\alpha D)}} - \frac{\left[q - q_0 e^{\frac{-b^2}{2}(1+\alpha D)t} \right]^2}{2\frac{\propto^2 \pi D}{b^2(1+\alpha D)}} \right) \right] dq \qquad (34)$$

$\because P_s \approx N(\mu, \delta^2)$ then we suppose that :

$$\mu = q_0 e^{\frac{-b^2}{2}(1+\alpha D)t} \qquad \text{and} \qquad \delta^2 = \sqrt{\frac{\alpha^2 \pi D}{b^2(1+\alpha D)}}$$

When we put above values in law of entropy [3] we get that :

$$S(t) = -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\delta^2}} e^{-\frac{(q-\mu)^2}{2\delta^2}} \left(-\frac{1}{2} Ln(2\pi\delta^2) - \frac{(q-\mu)^2}{2\delta^2} \right) dq$$

$$S(t) = \frac{1}{2} Ln(2\pi\delta^2) \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\delta^2}} e^{-\frac{(q-\mu)^2}{2\delta^2}} dq + \frac{1}{\sqrt{2\pi\delta^2}} \int_{-\infty}^{\infty} \frac{(q-\mu)^2}{2\delta^2} e^{-\frac{(q-\mu)^2}{2\delta^2}} dq$$

Put that $y = q - \mu$ $\quad q = y + \mu$ $\quad dy = dq$ then we have:

$$S(t) = \frac{1}{2} Ln(2\pi\delta^2) + \frac{2}{\sqrt{2\pi\delta^2}} \int_{0}^{\infty} \frac{y^2}{2\delta^2} e^{-\frac{y^2}{2\delta^2}} dy \qquad (35)$$

After performing the integral we find the value of entropy as:

$$S(t) = \frac{1}{2} Ln(2\pi\delta^2) + \frac{1}{\sqrt{\pi}} \Big\rceil \frac{3}{2} = \frac{1}{2} Ln(2\pi\delta^2) + 1 \qquad (36)$$

4. Conclusion

In this work, in one section we study the first moment, and we deduced some equation for the first moment, so after several derivations we have an equation of the fourth order, where we use MATLAB to solve this equation, as well as we assumed values for the constant and we have a table of the assumed values, and we also draw the first moment after assuming values for the constant. In two sections, we discussed some dynamic properties and extracted values for these properties through some derivations and integrations that we have done. Finally, we took the conclusion.

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