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Modeling and Analysis of Dynamic Loads on a (2D) Frame Using an MDOF System

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Abstract

Dynamic safety assessment of engineering structures under loads is an topic in structural engineering due to the complexity of energy transfer in multi degree of freedom (MDOF) systems. The main limitation of conventional analytical methods lies in their inability to accurately predict dynamic behavior, in addition to their computational cost, especially when analyzing transient responses. The equation of motion was used and solved numerically in MATLAB using the fourth-order Runge–Kutta (RK4) method. Different loading cases were applied by varying the load matrix while keeping the mass, stiffness, and damping matrices constant. The results showed that the proposed model is capable of accurately representing the dynamic response, as the time histories exhibited stable behavior during transient and steady state phases. Furthermore, the results indicated that variations in load magnitude and location influence displacement behavior, with an effect of dynamic coupling in energy transfer between nodes. The proposed model enhances prediction accuracy while maintaining computational efficiency, reduces the need for complex simulations, and provides a reliable tool for structural design.

Keywords: Dynamic analysis, Equations of motion, Multi-Degree-of-Freedom (MDOF), Runge-Kutta method, Two-Dimensional frame (2D).

نمذجة وتحليل تأثير الاحمال الديناميكية على أطار ثنائي الابعاد بأستخدام نظام متعدد درجات الحرية

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الخلاصة

يعد تقييم السلامة الديناميكية للهياكل الهندسية تحت تأثير الأحمال الديناميكية مهم في الهندسة الإنشائية، ذلك بسبب تعقيد انتقال الطاقة في الأنظمة متعددة درجات الحرية (MDOF). تكمن المحدودية الرئيسية للطرائق التحليلية التقليدية في عدم قدرتها على التنبؤ الدقيق بالسلوك الديناميكي، إضافةً إلى كلفتها الحسابية العالية، خاصةً عند تحليل الاستجابات العابرة. تم استخدام معادلة الحركة وحلّها عددياً في برنامج MATLAB باستعمال طريقة رينج–كوتا من الرتبة الرابعة (RK4). كما تم تطبيق حالات تحميل مختلفة من خلال تغيير مصفوفة الأحمال مع بقاء مصفوفات الكتلة والصلابة والتخميد ثابتة. أظهرت النتائج أن النموذج المقترح قادر على تمثيل الاستجابة الديناميكية بدقة عالية، إذ أظهرت الاستجابات الزمنية سلوكاً مستقراً خلال مرحلتي الاستجابة العابرة والمستقرة. علاوةً على ذلك، أشارت النتائج إلى أن التغيرات في مقدار الحمل وموقعه تؤثر بشكل مباشر في سلوك الإزاحة، مع وجود تأثير واضح للاقتران الديناميكي في انتقال الطاقة بين العقد. وبصورة عامة، يعزز النموذج المقترح دقة التنبؤ مع الحفاظ على الكفاءة الحسابية، ويقلل الحاجة إلى المحاكاة المعقدة، ويوفر أداة موثوقة للتصميم الإنشائي.

1. Introduction

Mathematical modeling is the process of using mathematical concepts to represent real-world phenomena and address important scientific and practical problems, in this process, a real-world scenario is converted into a mathematical formulation, analyzed using appropriate methods, and the results are then interpreted in the context of everyday life. To predict future outcomes, the solution of the mathematical model is translated back into real-life concepts and conditions [1].

Structural engineering is a major branch of civil engineering that focuses on the study, analysis, and design of structures such as buildings, bridges, and tunnels. Its main objective is to ensure the safety, stability, and good performance of structures under different types of loads, such as gravity, wind, and seismic forces. With rapid urban development, mathematical models have become very important for predicting structural behavior before construction [2].

Structural vibrations are considered an important and common phenomenon in engineering, with various applications aimed either at reducing these vibrations or utilizing them. In general, most human activities are affected directly or indirectly by the effects of vibrations [3].

Most complex engineering structures can be simplified by representing them as multi-degree of-freedom (MDOF) systems. These systems require advanced mathematical solutions because they consist of multiple interconnected masses that interact dynamically with each other. The study by Professor Anil K. Chopra in his well-known book Dynamics of Structures serves as a primary reference, explaining how to handle such systems and their applications in earthquake engineering and vibration analysis [2, 4].

The results in [5] show that simplified MDOF models can effectively represent the essential characteristics of structural vibration when validated with numerical simulations.

MDOF systems involve two or more independent coordinates to describe the motion of a structure. Continuous systems, which theoretically have an infinite number of degrees of freedom, are not included in this category. However, the widespread use of the finite element (FE) method allows structures that were once treated as continuous to be represented as MDOF systems, MDOF models now account for the majority of practical problems in structural dynamics [5, 6].

According to Kasimali in 2016, rigid frames refer to stable structural systems composed of elements linked by joints that resist motion. This study applied a two-dimensional (2D) planar frame with all structural members and applied dynamic loads in a single plane with horizontal motion only. It allows one to accurately analyze horizontal forces, the main drivers of the building's structural response [7].

This study aims to analyze the dynamic response of a 2D frame modeled as an MDOF system under various periodic load conditions using the RK4 numerical method.

2. Structural Loads

Structural loads can be broadly classified into four groups.

2.1 Dead Loads

Dead loads are structural loads that remain constant over time. They include the self-weight of structural elements such as walls, beams, and columns (e.g., concrete slabs and steel girders), in addition to permanent fixtures. Since the dimensions of these elements are initially determined based on architectural drawings before the final structural analysis, dead loads can be estimated with a relatively high degree of accuracy [8].

2.2 Live Loads

Live loads are variable in magnitude and position due to the use of a structure. Their design values are defined in building codes, and structural elements are designed for the load arrangement that produces the maximum stress. examples include occupants, furniture, and movable equipment [8].

2.3 Wind Loads

Wind loads are caused by air movement around a structure and are affected by location, nearby obstacles, structural geometry, and dynamics. Most building codes estimate them using the relationship between wind velocity and dynamic pressure q on exposed surfaces [9]. Mehta et al. [10] employed an indirect approach supported by extensive data to investigate the effect of wind speed, proposing a practical alternative method. The results showed that wind speed and direction have a significant and fundamental impact on the deflection of high-rise buildings [10].

2.4 Earthquake Loads

An earthquake is a sudden ground vibration. Horizontal motion, more than vertical, causes structural damage. As foundations move, the building's upper part resists inertia, creating horizontal vibrations and shear forces. Accurate stress prediction requires dynamic analysis of the building's mass and stiffness [9]. Earthquakes are considered a major natural hazard that threatens structural integrity and human safety [4, 11]. Therefore, accurate assessment of seismic behavior requires conducting dynamic analysis throughout all stages of a structure's lifecycle, including design, construction, and inspection [12-14]. Hallebrand and Jakobsson [15] analyzed high-rise buildings under static and dynamic loads, focusing on deflections, resonance frequencies, accelerations, and stability. They also examined finite element modeling techniques and compared vertical and horizontal loading effects using different modeling approaches.

The loads acting on the building are as shown in Figure 1.

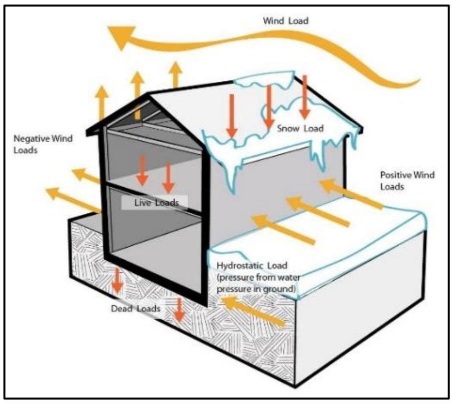

Figure -1 The effect of several types of loads on a building [16]

3. Subject

This study investigates the dynamic response of multi-degree-of-freedom (MDOF) structures under periodic loads, focusing on the interaction between key system components such as mass, stiffness, and damping. The study aims to develop accurate predictive models for the structural response, including the following time-dependent variables:

- Displacement: $x(t)$.
- Velocity: $\dot{x}(t)$.
- Acceleration: $\ddot{x}(t)$.

These variables are analyzed using advanced numerical algorithms to solve the governing equations of motion, enabling precise and efficient assessment of structural behavior.

4. Model

In this model, the equation of motion, where two nodes are used to describe the dynamic behavior of the structure [17-19]:

$$m\ddot{x}(t) + \mathrm{c}\dot{x}(t) + kx(t) = F(t) \tag{1}$$

Where m is the mass matrix of nodes, $\ddot{x}$ is the acceleration vector matrix of nodes, c is the damping coefficient matrix of nodes, $\dot{x}$ is the velocity vector matrix of nodes, k is the stiffness matrix of nodes, x is the displacements vector matrix of nodes and $F(t)$ is the applied external load vector matrix on nodes.

The structural system is represented by a node, which is a specific point within the structure used to describe the motion of a particular part of the building specifically, horizontal motion in this study. The movement of this node reflects how the structure responds to the applied loads and helps in understanding the effects of mass, stiffness, and damping on the overall structural response in a simple and clear manner.

Example of the system:

$$m = \begin{bmatrix} 500 & 0 \\ 0 & 500 \end{bmatrix} \,.\, c = 0.02k = \begin{bmatrix} 40 & -20 \\ -20 & 40 \end{bmatrix}$$

$$k = \begin{bmatrix} 2000 & -1000 \\ -1000 & 2000 \end{bmatrix} \,.\, F = \begin{bmatrix} 5000sin\,(2\pi t) \\ 0 \end{bmatrix}$$

Acceleration vector:

$$\ddot{x} = m^{-1}(F - c\dot{x} - kx)$$

$$m^{-1} = \begin{bmatrix} \frac{1}{500} & 0 \\ 0 & \frac{1}{500} \end{bmatrix} \; \ddot{x} = \left(\begin{bmatrix} 5000sin\,(2\pi t) \\ 0 \end{bmatrix} - \begin{bmatrix} 40 & -20 \\ -20 & 40 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x} \end{bmatrix} - \begin{bmatrix} 2000 & -1000 \\ -1000 & 2000 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right)$$

Displacement of the Node 1:

$$\ddot{x}_1 \ = \frac{1}{500}[5000\, sin(2\pi t) - (40\dot{x}_1 - 20\dot{x}_2) - (2000x_1 - 1000x_2)]$$

Displacement of the Node 2:

$$\ddot{x}_2 \ = \frac{1}{500}[0 - (-20\dot{x}_1 + 40\dot{x}_2) - (-1000x_1 + 1000x_2)]$$

Initial values:

$$x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \dot{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

5. Solution Methodology

- Define the structural properties, including mass, stiffness, and damping matrices.
- Formulate the equation of motion and convert them into a second-order system.
- Solve the system numerically using the Runge–Kutta method.
- Compute nodal displacements over time.
- Determine peak and steady-state responses.

6. Numerical Solution

Numerical analysis is a branch of mathematics that focuses on designing methods to obtain numerical solutions for complex problems, especially when finding analytical solutions is difficult or impossible. when dealing with a very large number of linear equations containing thousands of unknowns, numerical methods are required to obtain approximate solutions. With the advancement of digital computers, it has become possible to use accurate mathematical models that support the work of researchers and engineers in various fields [3, 20].

7. Mathematical Technique

This study examines the effect of applied loads (F) while keeping stiffness (K) and damping (C) constant, in order to analyze the relationship between the applied load and the structural response over time. The Runge–Kutta method was used in MATLAB to solve the second-order differential equations, which enabled an accurate representation of the time history of displacements during both the transient and steady-state response phases [19].

The solution was developed through several stages, which can be outlined as follows:

- Identify and understand the problem.
- Develop appropriate plans for the solution.

- Implement the solution according to the plan.
- Evaluate the accuracy and validity of the solution.
- Generalize the solution and apply it to new or unique problems [21].

8. Results and Discussion

Load distribution represents a fundamental basis in structural analysis in civil engineering, where its importance lies in verifying the consistency of predictive mathematical models with computational results [22].

Case 1: The dynamic response of the 2D frame under the same load applied on Node 1 in the above example, as shown in Figure 2.

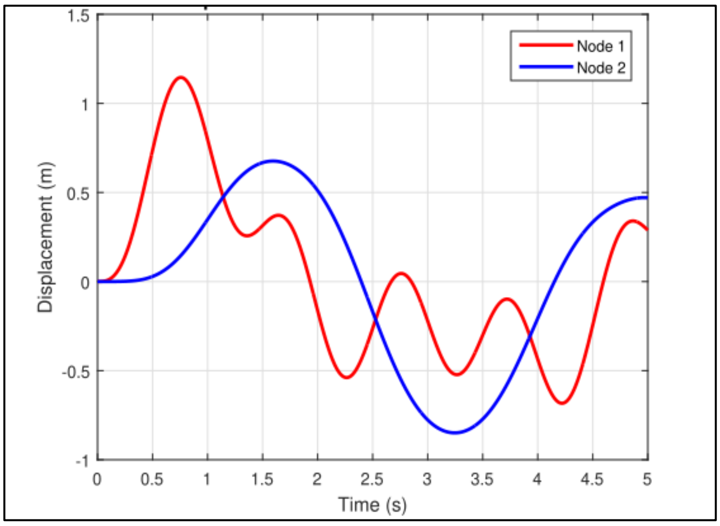

Figure -2 The time response of horizontal displacement in a 2D frame system under dynamic loading

Figure 2 shows that Node 1 (red) moves first with a larger amplitude due to the applied direct load as shown in Table 1, while Node 2 (blue) moves later with a smaller amplitude despite not being subjected to any external load, due to structural coupling through stiffness interaction. both nodes start from rest, and each node represents a single degree of freedom (horizontal displacement) at the floor level. his indicates that dynamic coupling alone is sufficient to induce motion in an unloaded node.

Table 1- Displacement response of Nodes 1,2 at maximum and minimum values under dynamic load

Node 1 (red)	In (0.8, 1.15) max. point	In (2.3, - 0.55) min. point
Node 2 (blue)	In (1.6, 0.6) max. point	In (3.25, - 0.8) min. point

Case 2: The dynamic response of the 2D frame under the same load applied on Node 2 $F = \begin{bmatrix} 0 \\ 3000\sin(2\pi t) \end{bmatrix}$, as shown in Figure 3.

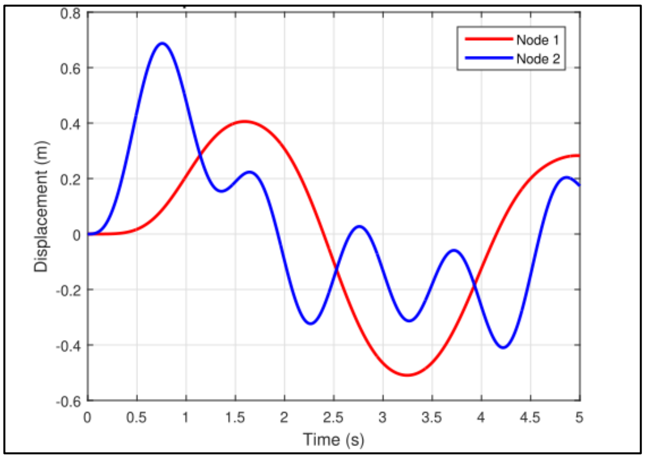

Figure -3 The time response of horizontal displacement in a 2D frame system under dynamic loading

Figure 3 shows that Node 1 (red) moves first with a smaller amplitude due to the absence of direct dynamic loading, while Node 2 (blue) responds later with a larger amplitude as a result of the applied dynamic load as shown in Table 2. the results indicate that both nodes are initially in a state of rest. the reduction in the applied load from 5000 N to 3000 N leads to a decrease in the overall amplitude, which is also reflected in the reduced response compared to Case 1. this confirms the linear behavior of the system and supports the validity of the proposed model.

Table 2- Displacement response of Nodes 1,2 at maximum and minimum values under dynamic load applied to Node 2

Node 1 (red)	In (1.6, 0.4) max. point	In (3.3, - 0.55) min. point
Node 2 (blue)	In (0.8, 0.65) max. point	In (4.2, - 0.42) min. point

Case 3: The dynamic force applied to Node 1 is removed to analyze the system response under no external load $F = \begin{bmatrix} (0)5000\sin\,(2\pi t) \\ 0 \end{bmatrix}$, as shown in Figure 4.

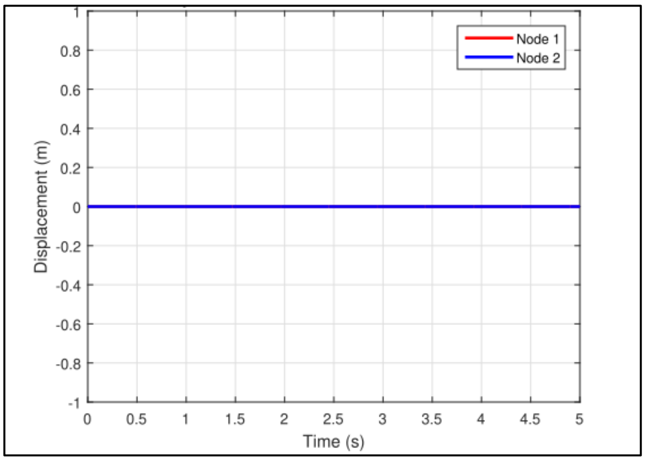

Figure -4 The time response of horizontal displacement in a 2D frame system under dynamic loading

Figure 4 shows that the displacements of both Node 1 and Node 2 are equal to zero as shown in Table 3, and the response appears as a straight line along the horizontal axis. this result is fully consistent with the mathematical model, which predicts that the system remains at rest in the absence of external loads and with zero initial conditions. Moreover, the absence of spurious oscillations or numerical drift confirms the accuracy of the computational model and the consistency in the formulation of both the mass and stiffness matrices.

Table 3- Displacement Response of Nodes 1,2 at maximum and minimum values in the absence of external dynamic loads

Node 1,2	In (0, 0) max. point	In (0, 0) min. point

Case 4: The dynamic force applied to Node 1 has been changed to a cos function, while Node 2 remains unloaded $F = \begin{bmatrix} 5000\cos(2\pi t) \\ 0 \end{bmatrix}$, as shown in Figure 5.

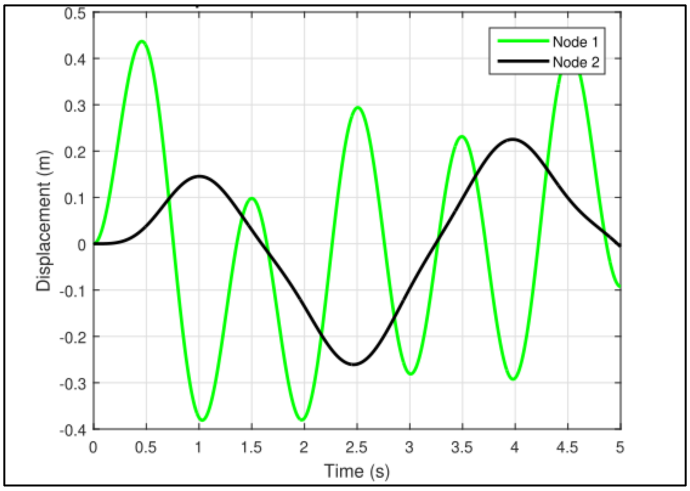

Figure -5 The time response of the horizontal displacement of the 2D frame nodes the first external dynamic load (cos)applied on Node 1

Figure 5 shows that Node 1 (green) exhibits a direct response to the applied sinusoidal force as shown in Table 4 characterized by high-amplitude oscillations. this energy is gradually transferred to the Node 2 (black) which responds with a smaller amplitude and slower oscillation due to its dynamic coupling with the Node 1. replacing the sin function with a cos function results only in a phase shift in time without altering the dynamic response of the system.

Table 4- Displacement response of Nodes 1,2 at maximum and minimum values under the effect of cos load applied to Node 1

Node 1 (green)	In (0.4, 0.44) max. point	In (1.05, - 0.38) min. point
Node 2 (black)	In (1.05, 0.15) max. point	In (2.45, - 0.2) min. point

Case 5: The dynamic force applied to Node 2 has been changed to a cos function, while Node 1 remains unloaded $F = \begin{bmatrix} 0 \\ 3000\cos(2\pi t) \end{bmatrix}$, as shown in Figure 6.

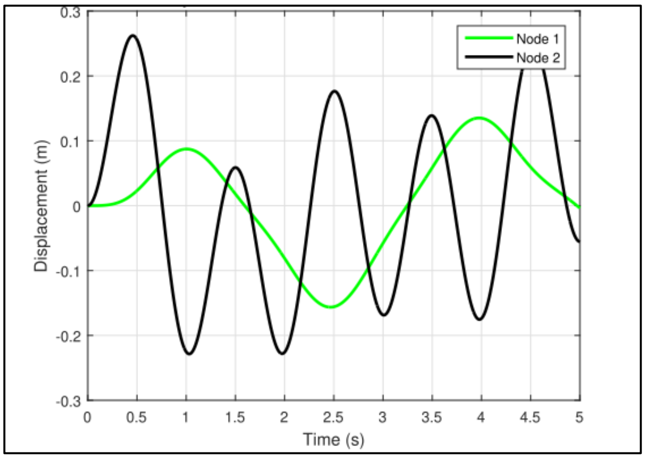

Figure -6 The time response of the horizontal displacement of the 2D frame nodes the first external dynamic load (cos) applied on Node 2

Figure 6 shows that Node 1 (green). It exhibits a smaller-amplitude response with slower oscillations due to its dynamic coupling with Node 2 (black) whereas Node 2 shows a direct response to the applied dynamic force as shown in Table 5, and is characterized by higher frequency oscillations. comparing this case with Case 2 confirms that switching from sine to cosine produces only a phase shift while preserving the same amplitude distribution pattern between the two nodes.

Table 5- Displacement response of Nodes 1,2 at maximum and minimum values under the effect of cos load applied to Node 2

Node 1 (green)	In (0.7, 0.12) max. point	In (2.4, - 0.16) min. point
Node 2 (black)	In (0.4, 0.25) max. point	In (1.1, - 0.2) min. point

Case 6: The dynamic effect of the force applied on Node 2 with Node 1 remaining unchanged $F = \begin{bmatrix} 5000\sin(2\pi t) \\ 3000\sin(2\pi t) \end{bmatrix}$, as shown in Figure 7.

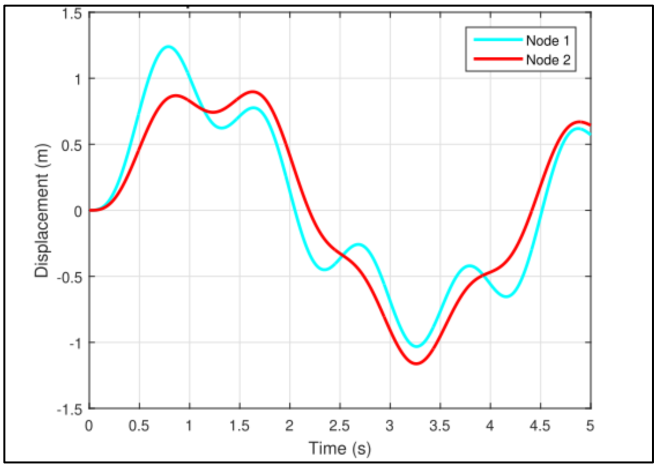

Figure -7 The time response of horizontal displacement in a 2D frame system under an external dynamic load applied to both Node 1 and Node 2

Figure 7 shows that Node 1 (cyan) reaches the maximum positive displacement, while Node 2 (red) reaches the maximum negative displacement as shown in Table 6, reflecting the flexible response of the structure. the load distribution across both nodes indicates a clear amplification in the structural response, where the displacement of Node 1 increased from 1.15 m in Case 1 to 1.25 m in this case. this reflects the additional effect of loading applied at Node 2, which is transmitted through dynamic coupling. the agreement between the model and the results highlights the importance of considering full nodal loading when designing structures subjected to seismic or wind forces.

Table 6- Displacement response of Nodes 1,2 at Maximum and Minimum Values Under the Effect of sin load applied to Nodes 1, 2

Node 1 (cyan)	In (0.8, 1.25) max. point	In (3.25, - 1.05) min. point
Node 2 (red)	In (1.6, 0.9) max. point	In (3.25, - 1.18) min. point

Case 7: The dynamic forces applied to Node 1 and Node 2 were changed to cos functions $F = \begin{bmatrix} 5000\cos(2\pi t) \\ 3000\cos(2\pi t) \end{bmatrix}$, as shown in Figure 8.

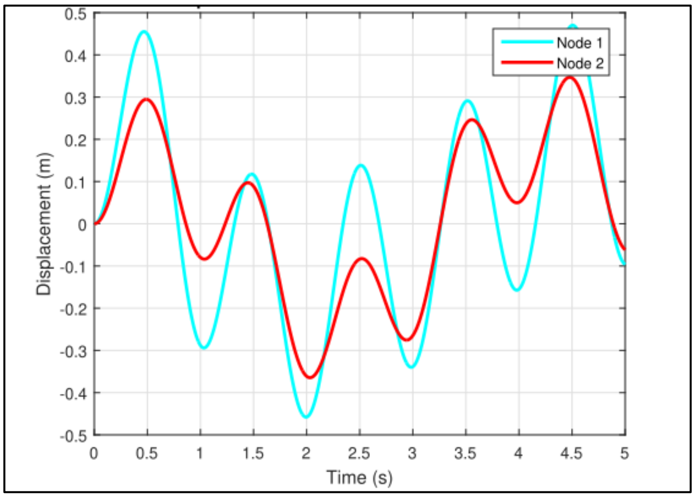

Figure -8 The time response of horizontal displacement in a 2D frame system under an external dynamic load applied to both Node 1 and Node 2

Figure 8 shows the effect of the periodic load on both nodes, where Node 1 (cyan) reaches larger positive and negative displacements compared to Node 2 (red) as shown in Table 7, reflecting the high flexibility of the structure when subjected to cosine (cos) functions. this results in continuous oscillatory motion between positive peaks and negative values over time. comparison with Case 6 confirms that replacing the sine function with a cosine function leads only to a phase shift, while the effect of simultaneous loading on both nodes remains evident through increased displacements compared to single-Node loading cases.

Table 7- Displacement response of Nodes 1,2 at maximum and minimum values under the effect of cos load applied to Nodes 1, 2

Node 1 (cyan)	In (0.45, 0.46) max. point	In (2.00, - 0.46) min. point
Node 2 (red)	In (0.5, 0.3) max. point	In (2.45, - 0.37) min. point

We will get the better results in Figure 4 under ideal static conditions because there are no external forces, the response of the first node (Node 1) coincides with that of the second node (Node 2) at zero throughout the entire time interval (from 0 to 5 seconds), indicating that the entire structure is in a stable and static equilibrium. the present case aims to validate the accuracy of the proposed numerical model. according to the equation of motion, when external loads are absent $F = 0$ and initial conditions are zero, the system remains in a state of complete equilibrium, consistent with what [2] indicated. the precise agreement of the numerical results with the zero axis confirms that the mass and stiffness matrices have been correctly formulated, in addition to the numerical stability of the fourth-order Runge-Kutta method, free from any programming errors or spurious oscillations.

9. Conclusion

The study reached the following conclusions:

1. Mathematical modeling: The results confirmed the accuracy and efficiency of the numerical solution based on the Runge–Kutta method in mathematical modeling using MATLAB for the analysis of the dynamic response of structural systems.

2. Analysis of excitation functions: Replacing the sine function (sin) with the cosine function (cos) in the dynamic loading results only in a phase shift in time, without affecting the peak displacement amplitudes or the overall dynamic behavior of the system.

3. Effect of load location: The structural response showed significant sensitivity to the point of load application. The node subjected directly to the load exhibited a larger displacement compared to other nodes, which responded subsequently due to system coupling.

4. Structural stability: The results show that, in the absence of external dynamic loads, the structure maintains a stable equilibrium state with no vibrations, as indicated by the zero response in the third case.

5. Comparison between cases: Comparing Case 1 with Case 6, the results indicate that simultaneous loading at both nodes led to an 8.7% increase in Node 1 displacement, rising from 1.15 m to 1.25 m. Furthermore, the comparison between Case 1 and Case 4 shows remained within a difference of less than 3% between both cases.

Statements on compliance with ethical standards and standards of research involving animals

This article does not contain any studies involving animals performed by any of the authors.

Disclosure and conflict of interest

The authors declare that they have no conflicts of interest.

References

[1] S. Banerjee, *Mathematical modeling: models, analysis and applications*. Chapman and Hall/CRC, 2021.

[2] J. Humar, *Dynamics of structures*. CRC Press, 2012.

[3] S. S. Rao, *Mechanical Vibrations*, Sixth ed. Pearson, 2018.

[4] A. A. Ahmed, M. Y. Fattah, and M. K. Mohsen, "Effect of frame foundation geometry on the dynamic response of high-speed turbo machine foundations," *Heliyon,* vol. 11, no. 1, p. e41050, 2025.

[5] E. Kausel, "Multiple Degree of Freedom Systems," in *Advanced Structural Dynamics*: Cambridge University Press, 2017.

[6] F. Yang, Y. Yang, and J. Wang, "Modeling and Validation of MDOF Structural Vibration and Penetration Dynamics: From Analytical Derivations to Finite Element Simulations," *Applied and Computational Engineering,* vol. 200, no. 1, pp. 111-120, 11/05 2025, doi: 10.54254/2755-2721/2026.KA28908.

[7] A. Kassimali, *Structural Analysis*. Cengage Learning, 2016.

[8] A. Kassimali, *Structural Analysis*, Fourth ed. Cengage Learning, 2011.

[9] D. Podder and S. Chatterjee, *Introduction to structural analysis*. CRC Press, 2021.

[10] K. Mehta, J. E. Minor, and T. Reinhold, "Wind speed-damage correlation in Hurricane Frederic," *Journal of Structural Engineering,* vol. 109, no. 1, pp. 37-49, 1983.

[11] R. Falcone, C. Lima, and E. Martinelli, "Soft computing techniques in structural and earthquake engineering: a literature review," *Engineering Structures,* vol. 207, p. 110269, 2020.

[12] J. A. Evans, R. R. Hiemstra, T. J. Hughes, and A. Reali, "Explicit higher-order accurate isogeometric collocation methods for structural dynamics," *Computer Methods in Applied Mechanics and Engineering,* vol. 338, pp. 208-240, 2018.

[13] J.-P. Noël and G. Kerschen, "Nonlinear system identification in structural dynamics: 10 more years of progress," *Mechanical Systems and Signal Processing,* vol. 83, pp. 2-35, 2017.

[14] G. Kerschen, K. Worden, A. F. Vakakis, and J.-C. Golinval, "Past, present and future of nonlinear system identification in structural dynamics," *Mechanical systems and signal processing,* vol. 20, no. 3, pp. 505-592, 2006.

[15] E. Hallebrand and W. Jakobsson, "Structural design of high-rise buildings," MSc Thesis, Lund University, Sweden, 2016.

[16] A. Khodadadi, "Basic concepts of structural design for architecture students," DXOpen: Open Educational Resources., 2022. [Online]. Available: https://archives.pdx.edu/ds/psu/36303

[17] I. Okawa, Y. Ishiyama, and M. Watabe, "Fundamentals of Structural Dynamics," 2010.

[18] T. Wang, H. Li, M. Noori, R. Ghiasi, S.-C. Kuok, and W. A. Altabey, "Seismic response prediction of structures based on Runge-Kutta recurrent neural network with prior knowledge," *Engineering Structures,* vol. 279, p. 115576, 2023.

[19] S. K. Sahani, "Runge-Kutta-Based Dynamic Simulation of a Multi-Degree-of-Freedom Vibrating System," *International Journal of Intelligent Systems and Applications in Engineering,* vol. 7, no. 4, pp. 275–284, 2019.

[20] B. M. Ayyub and R. H. McCuen, *Numerical analysis for engineers: methods and applications*. Chapman and Hall/CRC, 2016.

[21] A. Arseven, "Mathematical Modelling Approach in Mathematics Education," *Universal Journal of Educational Research,* vol. 3, no. 12, pp. 973-980, 2015.

[22] A. Sil, "mathematical modelling and structural analysis in civil engineering: a comprehensive research study on predictive methods for load distribution, stress behaviour, and design optimisation," *International Journal of Applied Mathematics,* vol. 38, no. 4s, pp. 1244-1254, 2025.