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Cost and Reliability Optimization by Using the Grey Wolf Algorithm

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Abstract

This study shows how to use the Grey Wolf Optimizer (GWO) to create a multi-objective reliability-cost optimization framework for complex networks. The goal is to determine the optimal reliability levels for each item so that the system as a whole is as reliable as possible and stays within budget. Two cost models, quadratic and exponential, are used to look at how making things more reliable will affect the economy. This work not only improves the computer optimization process but also adds important theoretical results that strengthen the mathematical foundation of the reliability allocation problem. It is shown that system reliability always increases as component reliabilities increase, and that the associated cost functions are always increasing and convex. So, the best allocations happen at the edge of the zone where reliability is possible. A similar analytical conclusion shows that the exponential cost model has a substantially larger marginal penalty at high reliability levels, which is why it costs more. The suggested architecture is used in a complex network with 15 components. The results reveal that the GWO algorithm finds the best balance between reducing costs and improving reliability. The exponential cost model has a greater marginal penalty near high reliability levels. This makes the quadratic model better suited to systems that require very high reliability, while the exponential model may be sufficient for systems that need intermediate reliability. Combining metaheuristic optimization with rigorous theoretical analysis provides a better understanding of the trade-offs between reliability and cost, helping us make better decisions for complex engineering systems. The results show that the Grey Wolf Optimiser (GWO) is a reasonable compromise between reliability and cost, with a system reliability of 0.9626 in a 15-component network. The analysis indicates that the exponential model is more expensive at high reliability levels than the quadratic model, and the quadratic model is more effective for high-reliability systems.

Keywords: Grey Wolf Optimizer; reliability allocation; reliability–cost tradeoff; convex cost models and complicated networks.

تحسين متعدد الأهداف للموثوقية والتكلفة باستخدام خوارزمية الذئب الرمادي

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الخلاصة

توضح هذه الدراسة كيفية استخدام محسن الذئب الرمادي (GWO) لإنشاء إطار تحسين متعدد الأهداف للموثوقية والتكلفة للشبكات المعقدة .الهدف هو تحديد مستويات الموثوقية المثلى لكل عنصر بحيث يكون النظام ككل موثوقًا قدر الإمكان ويظل ضمن الميزانية .يتم استخدام نموذجين للتكلفة، التربيعي والأسّي، للنظر في كيفية تأثير جعل الأشياء أكثر موثوقية على الاقتصاد . لا يحسن هذا العمل عملية تحسين الكمبيوتر فحسب، بل يضيف أيضًا نتائج نظرية مهمة تعزز الأساس الرياضي لمشكلة تخصيص الموثوقية .يُظهر أن موثوقية النظام تزداد دائمًا مع زيادة موثوقية المكونات، وأن دوال التكلفة المرتبطة دائمًا ما تكون متزايدة ومحدبة .لذا، فإن أفضل التخصيصات تحدث عند حافة المنطقة التي تكون فيها الموثوقية ممكنة .تظهر نتيجة تحليلية مماثلة أن نموذج التكلفة الأسية لديه عقوبة هامشية أكبر بكثير عند مستويات موثوقية عالية، ولهذا السبب يكلف أكثر .يتم استخدام الهيكل المقترح في شبكة معقدة تحتوي على 15 مكونًا .تكشف النتائج أن خوارزمية GWO تجد التوازن الأفضل بين تقليل التكاليف وتحسين الموثوقية .نموذج التكلفة الأسية لديه عقوبة هامشية أكبر بالقرب من مستويات الموثوقية العالية .هذا يجعل النموذج التربيعي أكثر ملاءمة للأنظمة التي تتطلب موثوقية عالية جدًا، بينما قد يكون النموذج الأسي كافيًا للأنظمة التي تحتاج إلى موثوقية متوسطة .دمج تحسين الخوارزميات الجينية مع التحليل النظري الدقيق يوفر فهماً أفضل للتوازنات بين الموثوقية والتكلفة، مما يساعدنا على اتخاذ قرارات أفضل للأنظمة الهندسية المعقدة.

1. Introduction

Reliability of complex systems is a key objective for evaluating system performance and availability in an uncertain, adverse environment [16]. Conventional analytic approaches often fail to handle reliability-enhancement problems when dealing with a large search space, interdependent components, and combinatorial constraints; hence, contemporary studies have turned to metaheuristic optimization algorithms to provide high-quality feasible solutions [1-2].

The GWO is a population-based imitation algorithm introduced by Mirjalili et al., which emulates the hierarchical command structure and collective predation of grey wolves. The minimal yet highly attractive combination of mathematical functions that model the surroundings and the chasing-and-catching operations, along with dynamically adjusting exploration, makes it easy to design and implement with fewer control parameters [3-17].

In the context of reliability optimization problems, the construction or allocation of systems can be formulated as constrained optimization tasks: for instance, the maximization of system reliability under constraints on cost, number of elements, energy flow rates, or time-in-operation [14]. GWO algorithms and their variant have been effectively used for such problems by casting the system configurations (e.g., component selections, redundancies, topology parameters) as decision variables and formulating objective (or multi-objective) functions that include reliability and cost or resource penalty terms [2-15]. Empirical evidence shows that GWO-based methods are capable of discovering trustworthy, cost-effective configurations in massive, realistic settings [2].

Besides, numerous improvements, combinations, and modifications of GWO have been introduced since its inception. Such attempts are intended to accelerate convergence, prevent premature convergence, and address large or high-dimensional problems in the algorithm [16]. E.g., adaptive and multi-strategy methods have been reported to be more effective on benchmarks and real-world engineering problems [4]. Such findings demonstrate the flexibility and adaptability of GWO in solving strength-based optimization problems.

For this purpose, the GWO can be used as a compact, robust tool to enhance the robustness of complex systems. When combined with appropriate reliability models (such as probabilistic or fault-tree models) and well-designed hybridization techniques, the GWO can produce high-quality, real designs in the face of complex, real-world constraints, making it an even more appealing tool for today’s engineering software.

Research Gap: Even though metaheuristic algorithms are often used to improve dependability and reduce costs in complex systems, most research to date focuses only on how well they perform computationally and through heuristics. Not much thought has been devoted to the mathematical structure of the optimization problem, especially the theoretical rules governing the trade-off between cost and reliability. In particular, the monotonic relationship between system reliability and component reliabilities, the boundary characteristics of optimal allocations, and the comparative marginal impacts of various cost models require further examination. So, there is still no solid theoretical framework that explains why optimal solutions come about and how cost models affect allocation behavior.

2. Methodology

This paper details the methodology employed to optimize reliability and cost using dual exponential and quadratic models and the Grey Wolf Optimizer (GWO).The goal is to

determine optimal component reliability values to maximize system reliability while minimizing cost. By using the following functions:
$F_1 = 50r^2$, $F_2 = 50e^r$

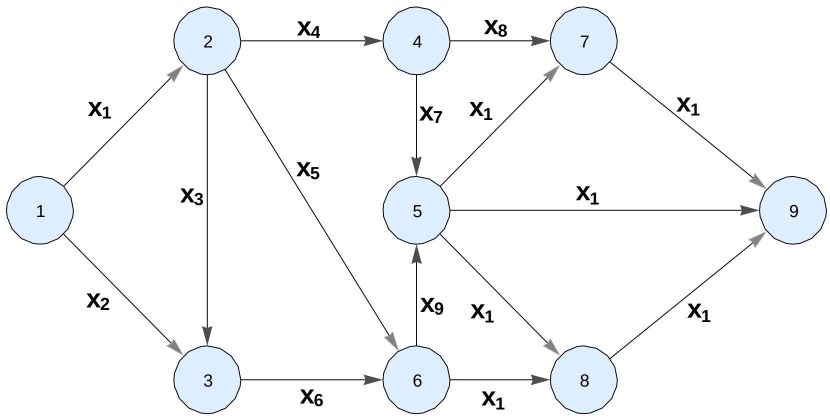

Figure-1 Complex network

2.1 Optimization using Grey Wolf Optimizer (GWO)

The concept of the proposed Grey Wolf (GW) optimizer is inspired by simulating the structure and hunting behavior of a grey wolf pack, in which the top-ranking members, called alpha, beta, and delta, are followed by other lower-ranking members. The hunting, surrounding, and attacking strategies are simulated, with the ability to construct the best solution in the search space [3-15]. GWO has the following advantages: it is easy to implement, has a strong ability to avoid stagnation points, and achieves high efficiency in dealing with multi-objective problems.

2.2 Theoretical Properties of the Optimization Problem

2.2.1 Proposition 1

Let the system reliability be an increasing function of component reliabilities $\boldsymbol{R_i}$, and let the cost functions $\boldsymbol{C_i R_i}$ be strictly increasing and convex on $\boldsymbol{0 < R_i < 1}$. Then any feasible increase in a component reliability improves system reliability at the expense of increasing total cost.

Proof

From Equations:

$$\mathbf{R_s = \prod_{i=1}^{n} R_i}$$

$$\mathbf{R_s = 1 - \prod_{z=1}^{p} \left(1 - \prod_{j=\alpha}^{\omega} R_j\right)}$$

system reliability $\boldsymbol{R_s}$ is monotonically increasing in each $\boldsymbol{R_i}$.

For the quadratic model:

$$\boldsymbol{C_i(R_i) = a_i R_i^2}$$

$$\boldsymbol{\frac{dC_i}{dR_i} = 2a_i R_i > 0}$$

For the exponential model:

$$C_i(R_i') = a_i e^{R_i}$$
$$\frac{dC_i}{dR_i} = a_i R_i e^{R_i} > 0$$

Thus cost strictly increases with reliability.

Therefore increasing reliability improves R_s but increases cost.

Theorem 1 (Boundary Optimal Allocation)

Consider the optimization problem:

$$\min C(R_1, \dots, R_n)$$

subject to

$R_s \geq R_G$, $0 \leq R_i < 1$

where $C_i(R_i)$ are convex increasing functions.

Then the optimal solution occurs on the boundary of the feasible reliability region, i.e., the optimal allocation satisfies:

$$R_S > R_G$$

Proof

Assume an optimal solution R^* satisfies:

$$R_S(R^*) > R_G$$

Since R_S is increasing in each R_i, there exists a small decrease ε in some R_K such that:

$$R_S(R^* - \varepsilon e^k) \geq R_G$$

Because $C_k(R_k)$ is increasing:

$$C(R^* - \varepsilon e^k) < \mathrm{C}(R^*)$$

This contradicts optimality.

Hence optimal solution must satisfy:

$R_S = R_G$

2.3 Optimization of Complicated Systems

Consider a complicated system with components interconnected in terms of reliability [9-13]. Employ the offered notes.

$$\mathrm{R_s}: \text{System's reliability}$$

$$0 \leq \mathrm{R_i} \leq 1 : \mathrm{i} = 1,2, \dots, 15;$$

$\mathrm{C_i}(\mathrm{R_i})$ costs of element i;

$\mathrm{C}(\mathrm{R_1}, \dots, \mathrm{R_n}) = \sum_{i=1}^{n} \mathrm{a_i c_i}(\mathrm{R_i})$: total system costs , $(\mathrm{a_i} > 0)$;

$\mathrm{R_G}$= System reliability Objective

The system's distinct modular designs and the unique functionalities of its components yield a range of possible outcomes. A collection of system components provides equivalent functionality, each with a different degree of reliability. The primary goal is the system's ability to effectively allocate resources to individual components or all components. Non-linear programming requires tackling problems [14-17]. Despite its non-linearity, the limitation serves a functional purpose and incurs costs that need analysis.

$$\mathrm{Min}\ \mathrm{C}(\mathrm{R_1}, \dots, \mathrm{R_n}) = \sum_{i=1}^{n} \mathrm{a_i C_i}(\mathrm{R_i}),\ \mathrm{a_i} > 0, \qquad (1)$$

Subjected in: $R_G \leq R_S$

$$0 \leq R_i < 1, \qquad \text{where } i = 1, \dots, n \qquad (2)$$

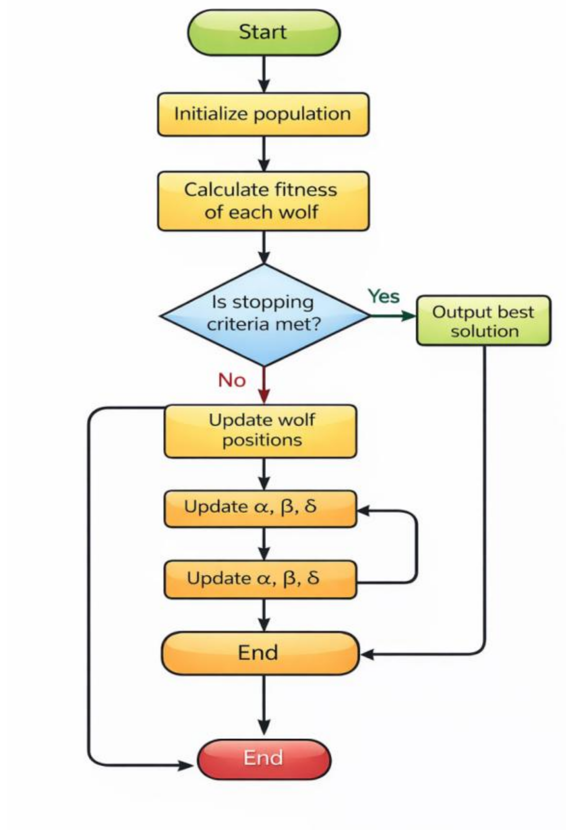

Figure -2 Grey Wolf Algorithm

2..3.1 Complex system implementation

To assess the complex system, it is important to reconfigure it into a more tractable network, akin to converting a series of entities into a parallel network. For series and parallel networks of n elements, the dependability is, respectively, [7, 18]:

$$R_s = \prod_{i=1}^{n} R_i \qquad (3)$$

$$R_s = 1 - \prod_{i=1}^{n}(1 - R_i) \qquad (4)$$

In this instance, R_s denotes the reliability network, indicating the component's reliability [6,8]. A comparison will be conducted between the reliability of each complex network and the minimum paths offered through the equations.

$$R_s = 1 - \prod_{z=1}^{p}\left(1 - \prod_{j=\alpha}^{\omega} R_j\right) \qquad (5)$$

The reliability function or polynomial consists of 519 terms it was calculated using the 16 minimum paths of the complex network and the 28 minimum cuts:

$$R_s = R_1R_2R_3R_6R_{13}R_{15} + \cdots + R_1R_2R_3R_3R_4R_5R_6R_7R_8R_9R_{10}R_{11}R_{12}R_{13}R_{14}R_{15} \qquad (6)$$

2.4 Quadratic behavior model

The a_i and b_i, c_i constant , where assume , $0 \leq R_i < 1$, where $i = 1,2, \dots , n$ suggested on the below formal:

$$\mathrm{C_i(R_i) = a_i 50\ r^2} \quad , \quad \mathrm{a_i} > 0 \qquad (7)$$

2.5 Exponential behavior model

The a_i and b_i Constant , where assumes, where $0 \leq R_i < 1$, where $i = 1,2, \dots, n$

$$\mathrm{C_i(R_i) = a_i 50\ e^r} \quad , \quad \mathrm{a_i} > 0 \qquad (8)$$

2.6 Theoretical Comparison Between Cost Models

Lemma 2

Assume for each component i that $0 < R_i < 1$ and $a_i > 0$. Consider the two cost models used in the paper:

$C_{1,i}(R_i) = 50a_iR_i^2$, $C_{2,i}(R_i) = 50a_ie^{R_i}$

Then the following statements hold:

1. Dominance:

$C_{2,i}(R_i) > C_{1,i}(R_i)$ for all $0 < R_i < 1$

2. Stronger marginal penalty near high reliability:

$$\frac{dC_{2,i}}{dR_i} < \frac{dC_{1,i}}{dR_i} \text{ for all } R_i \geq \ln(2)$$

3. Convexity dominance (risk of “cost explosion”):

$$\frac{d^2C_{2,i}}{dR_i^2} > \frac{d^2C_{i,1}}{dR_i^2} \ \ for\ all\ \ 0 < R_i < 1$$

Proof

(1) Dominance

For $0 < R < 1$ we use the inequality $e^R \geq 1 + R$ (true for all real R). Then:

$$e^R - R^2 \geq (1 + R) - R^2 = 1 + R - R^2$$

Define $g(R) = 1 + R - R^2$. On (0,1),

$$g'(R) = 1 - 2R, \qquad g''(R) = -2 < 0,$$

so g is concave and attains its minimum at the boundary. Since

$$g(0) = 1 > 0, \qquad g(1) = 1 > 0,$$

we get $g(R) > 0$ for all $0 < R < 1$. Hence $e^R - R^2 > 0$, then:

$$50a_ie^{R_i} > 50a_iR_{2,i} \Rightarrow C_{2,i}(R_i) > C_{1,i}(R_i)$$

(2) Marginal penalty near high reliability)

First derivative:

$\frac{dC_{1,i}}{dR_i} = 100a_iR_i$, $\frac{dC_{2,i}}{dR_i} = 50a_ie^{R_i}$

We need

$$50a_ie^{R_i} > 100a_iR_i \Leftrightarrow e^{R_i} > 2R_i.$$

So $e^R \geq 2$, when $R \geq ln2$, and because $2R \leq 2$, when $R \leq 1$, so , $\forall R \in [ln2,1)$:

$e^R \geq 2 \geq 2R \Rightarrow e^R > 2R,$ then , the result was prove.

(3) Convexity dominance

Second derivatives

$\frac{d^2C_{2,i}}{dR_i^2} = 100a_i$, $\frac{d^2C_{i,1}}{dR_i^2} = 50a_ie^{R_i}$

Since $e^{R_i} > 1$ on (0,1) , we get :$50a_i < 50a_ie^{R_i}$

Then , $\frac{d^2C_{2,i}}{dR_i^2} > \frac{d^2C_{i,1}}{dR_i^2}$ ■

Table 1. Summary table for R_I, C_I and Best value R_N and C_N by GWA with F_1 and F_2

Components	Optimized Reliability	Cost C1	Cost C2
R_1	0.9900	49.0050	134.5617
R_2	0.9573	45.8234	130.2358
R_3	0.5000	12.5000	82.4361
R_4	0.7246	26.2504	103.1926
R_5	0.5955	17.7332	90.7002
R_6	0.9875	48.7590	134.2274
R_7	0.5896	17.3830	90.1657
R_8	0.5682	16.1403	88.2508
R_9	0.9341	43.6309	127.2512
R_{10}	0.6519	21.2484	95.9588
R_{11}	0.5000	12.5000	82.4361
R_{12}	0.8226	33.8345	113.8218
R_{13}	0.6885	23.7039	99.5397
R_{14}	0.6682	22.3252	97.5371
R_{15}	0.8996	40.4600	122.9255

System Reliability: 0.9626

3. Result and Discuses

By locking a table 1 the following notes can be recorded. Reliability and Cost Analysis Results

1. Highest Reliability Values for Components:

· Component R_1: 0.99· Component R_2: 0.9573· Component R_6: 0.9875 , R_9: 0.9341

2. Lowest Reliability Values for Components:

Component R_3: 0.5000, Component , R_8: 0.5682, R_7: 0.5896 R_{11}: 0.500

3. Cost of High-Reliability Components (R_1): Cost C_1: 49.0050 ,

Cost C_2: 134.5617

4. Cost of Less Reliable Components (R_3): Cost C_1: 12.500 , · Cost C_2: 82.4361

5. System Reliability: $R_s = 0.9626$

6. Total System Cost: Cost C_1: 431, 2971 , Cost C_2: 1593.2404

Fig.3 Shows the values of R_i and Fig.4 shows the values C_1 and C_2.

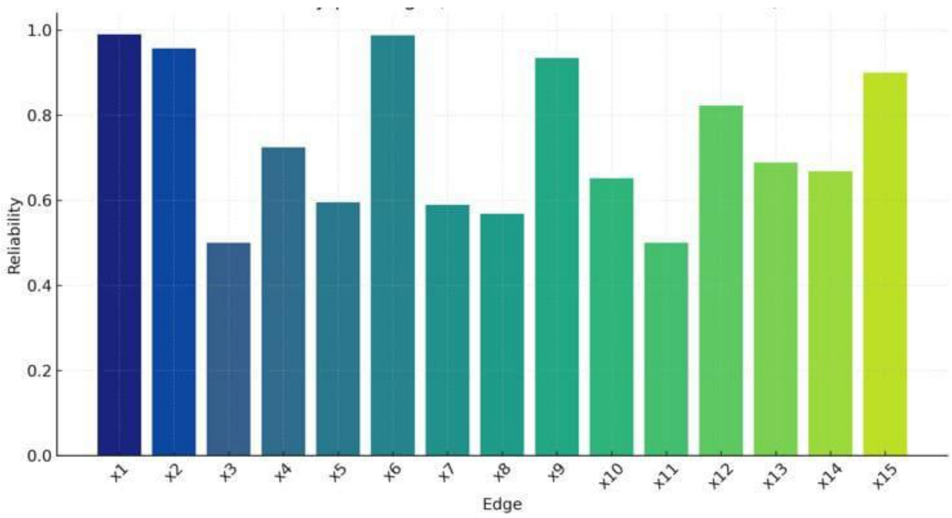

Figure -3 Values of R_I

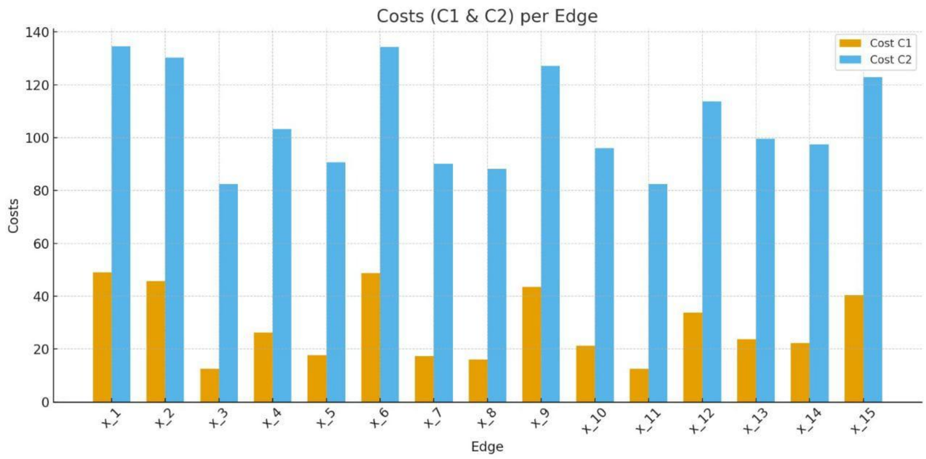

Figure -4 Values of C_i

4. Conclusion

This paper presented a multi-objective optimization model based on the Grey Wolf Optimizer (GWO) to achieve a reliability-cost trade-off in complex systems. The findings showed that the proposed method is effective in determining optimal reliability allocation within budget constraints, achieving a system reliability of 0.9626 with 15 components in the network. The analysis has validated that the exponential cost model yields much higher costs at high reliability levels, while the quadratic model offers a more efficient solution for highly reliable systems. These results indicate the suitability of the proposed method in aiding decision-making in the design of complex engineering systems.

Future work may extend this framework to more complex or uncertain reliability models.

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Ethical responsibilities of authors

The author declares that this manuscript is original, has not been published before, and is not under consideration for publication elsewhere. The author confirms that no data have been fabricated or manipulated, and that all sources have been properly cited.

Statements on compliance with ethical standards and standards of research involving animals

This article does not contain any studies involving animals or human participants performed by the author.

Disclosure and conflict of interest

The author declares that there are no conflicts of interest

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