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A Simulation Investigation of the Effect of Polluted Observations on Zero-Inflated Poisson Regression

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Abstract

Bulk numbers are popular for countable dataries, such as inflated zero regression prototypes. Zero-truncation operations with zero truncation can be effective in elaborating problems with prototypes numbers of zeros, as the behaviour of zeros in the datarie s shown may elicit difficulties, when they are solved by zero-truncation operations on the databases. The different Poisson threshold prototypes that use are known as the smallest value that gives the shift of the distribution from the normal regression to the zero inflated regression. This research includes simulated experiments according to the difference in the sample size ($n = 20,50,100$) and the initial values of the Polluted number ($\lambda = 1,2,3$), inflation parameter ($P = 0.1,0.3$) and (Maximum Likelihood Estimation (MLE), Method of Moment Estimation (MME)) for ZIP with (1000) iterations. The simulation results showed that the inflated zero parameter was affected with (sample size, the distribution parameter value and the initial value of the inflated zero parameter). Other estimation methods can be adopted, including Shrinkage Bayesian and Zero-Inflated Binomial regression models.

Keywords: Poisson Distribution, Zero Amplified Poisson Model, Zero inflated Regression, Maximum Likelihood Estimation, Method of Moment Estimation, Criteria of Minimum Absolute Different, Mean Square Error Criteria.

دراسة المحاكاة لتآثير الملاحظات الملوثه عل انحدار بوسون الصفري

فاطمة كميت عبدالله

قسم هندسة الموارد المائية, كلية الهندسة, جامعة بغداد, بغداد, العراق

الخلاصة

تمتلك البيانات المتجمعه الانتشار الكبير في البيانات القابلة للعد ومنها نماذج الانحدار الصفري المتضخم ويمكن لهذه النماذج ان تمتلك الفاعالية الكبيرة لحل المشكلات الخاصه بتضخم الاعداد الصفرية حيث ان سلوك الاصفار ضمن البيانات عند عمليات الاقتطاع يشكل مشاكل متزايده وان حل النماذج التي تمتلك الاصفار المتضخمه يمكن ان يتم من خلال الاقتطاع الصفري وهذا مايسبب المشاكل . تُعرف نماذج عتبة بواسون المختلفة المستخدمة بأنها أصغر قيمة تعطي تحول التوزيع من الانحدار الطبيعي إلى الانحدار المضخم الصفري. يشتمل هذا البحث على تجارب محاكاة تعمتد التغير في حجم العينة (n=20,50,100) والقيم الأولية للعدد الملوث (==1,2,3) ومعلمة التضخم (P=0.1,0.3) و(الحد الأقصى) طريقة الامكان الاعظم (MLE)، طريقة تقدير العزوم (MME)) مع (1000) تكرار. أظهرت نتائج المحاكاة تأثر مقدر معامل الصفر المضخم بكل من (حجم العينة، قيمة معلمة التوزيع، والقيمة الأولية لمعلمة الصفر المضخم). يمكن اعتماد طرق تقدير أخرى، بما في ذلك نماذج الانحدار الافتراضية ذات الحدين المنكمشة والصفرية المضخمة.

1. Introduction

The study of statistical models is crucial for understanding and predicting complex phenomena, particularly in the field of data analysis and regression modelling. One such area of interest is the Zero-Inflated Poisson Regression (ZIPR), a statistical method commonly used to analyse count data characterized by an excess of zero values. However, in real-world applications, datasets are often susceptible to the presence of polluted or contaminated observations, which may significantly impact the reliability and accuracy of statistical models[1].P. Banerjee and et al presented new suggestions to improve the work of LASSO methods by adapting weights to the data, with the result that these methods demonstrated excellent ability to determine the final mode[2]. H. Naya and et al compared four ZIP models (Poisson and Zero) to analyze dark areas in the wool fibers of sheep by applying real data with simulation data[3].S. Saffari and R. Adnancompared the effect of estimating Zero-inflated Poisson regression model parameters by comparing complete data and censoring data[4].C. Fengevaluated and compared the performance of the zero-inflated and hurdle models by generating and processing simulation data for each[5]. D. Lambertused real data and simulations that depict defects in manufacturing to demonstrate the effectiveness of the ZIP model in treating them through the use of the best possible method[6].Classical estimation methods: maximum likelihood and moment estimation methods are considered among the most widely applied methods for estimating parameters for distribution systems and others. Many researchers have relied on them to estimate parameters. In addition nonparametric estimation of functions and parameters has taken an important role among researchers in applications of real data studies and simulations[7], [8], .

This research aims to investigate the impact of polluted observations on the performance and robustness of Zero-Inflated Poisson Regression models. Polluted observations refer to data points that deviate significantly from the underlying assumptions of the model, potentially introducing biases and distorting the parameter estimates. This research includes simulation experiments according to the difference in sample size and initial values of the pollution parameter for inflation difference factor with two estimation methods.

The following section includes the following (ZAPM), the third section deals with the topic of (ZIR), while the fourth section includes the MLE and MOM methods for estimating ZIP parameters, and finally the fifth and sixth sections include simulation experiments and their results.

2. Zero Amplified Poisson Model (ZAPM)

This model is based on a zero inflation condition according to the Poisson distribution, According to the following probability mass function [3,4].

$$Pr\ (X = 0)\ = P + (1 - P\ )e^{-\lambda}, \lambda > 0 \tag{1}$$

$$Pr\ (X = x_i)\ =\ (1 - P\ )\frac{\lambda^{x_i} e^{-\lambda}}{x_i!},\ x_i = 1,2,\cdots \tag{2}$$

The Poisson process of observation (i) should account for the expectation parameter of the random variable (x_i) when any positive integer greater than zero corresponds to the Poisson parameter of the Poisson process of observation (i) as the probability distribution relies on the positive integer.

The symbol (P) denotes the inflated zero probability, which refers to the probability of an operation resulting in a variable value equal to zero. The expected value is[9]:

$$\mu_x = \sum x(1-P)\frac{\lambda^x e^{-\lambda}}{x!} = \lambda(1-P)\sum_x \frac{\lambda^{(x-1)}e^{-\lambda}}{(x-1)!} \tag{3}$$

The mean will be:

$$\mu_x = (1-P)\lambda \tag{4}$$

The variance will be:

$$var(x) = E(X^2) - (E(X))^2$$

$$E\,(x^2) = E\big(x(x-1)\big) + E(x)$$

$$E\,\big(x(x-1)\big) = (1-P)\lambda^2 \sum_x x(x-1)\frac{\lambda^{x-2}e^{-\lambda}}{x(x-1)(x-2)!} = (1-P)\lambda^2$$

$$E\,(x^2) = (1-P)\lambda^2 + (1-P)\lambda = (1-P)\lambda(1+\lambda)$$

$$var(x) = (1-P)\lambda(1+P\lambda) \tag{5}$$

3. Zero inflated Regression (ZIR)

Inflated zero regression models are widely used for countable data and particularly popular. However, the presence of zeros in the observed data can pose challenges for these models. The number of zeros and non-zeros can be generated through zero-truncation operations, which involve removing the zeros from the data. The various Poisson threshold models that are used are referred to as these holding. This term represents the smallest value that causes a shift in the distribution from a normal regression to a zero-inflated regression.

Let's consider a discrete random variable (X ∈ N) that represents the number of occurrences in a given experiment. Additionally, let (C) be an indicator that takes a value of either 1 or 0 for a latent category within the conditional distribution. With these considerations, we can define the Zero-inflated Poisson Regression model (ZIPRM) as follows:

$$\frac{Y}{C} = c \sim p(y;\mu.\vartheta), c = 0{,}1$$

The probability mass function ($p(y;\mu.\vartheta)$) denotes a function with parameters (μ,ϑ), and there exists an extra parameter known as the heterogeneity parameter, which can be present in the negative binomial regression model. The variable (Y) represents the marginal distribution, making it the dependent variable, and thus the marginal distribution is (Y) [3,4].

$$f_Y(y;\mu.\vartheta) = p(C=1)p\left(Y=\frac{y}{C}=1\right) + p(C=0)p\left(Y=\frac{y}{C}=0\right) \tag{6}$$

$$let\ P = p(C=0)$$

$$f_Y(y;\mu.\vartheta) = P\ I\,\{y=0\} + (1-P\,)p_Y(y;\mu.\vartheta) \tag{7}$$

The logarithm of the join function$\mu_0 = Ne^{x^T\beta}$would be:

$$Log(\mu_0) = Log(N) + x^T\beta \tag{8}$$

$$P(\alpha) = h^T(\omega^T\alpha)$$

$$\mu(\beta) = g^T(x^T\beta) \tag{9}$$

- ($P(\alpha)$) is a zero-inflated regression parameter and represents the probability corresponding to zero-inflation.
- (α) is the parameter of distribution.
- ($\mu(\beta)$) is the average of the counts in the zero regression is a function of (β).
- ($Log(N)$) is the estimator for ($\hat{\alpha_0}$)

4. Estimation of ZIP Parameters

Suppose we have observations of size (n), which are $(X_1, X_2, \cdots, X_n)$, so that they are all independent and identically Zero-inflated Poisson distribution with (P, λ) parameters that should be estimated.

4.1 Maximum Likelihood Estimation (MLE) for (ZIP)

Let $\widetilde{X} = (X_1, X_2, \dots, X_n)$ be a sample with size (n) distributed as (ZIP) with (P, λ) parameters. Then the likelihood function will be [10,11,12]:
With (Y) represent number of (0) values in (X_i) values, then:

$$L\left(P, \frac{\lambda}{\widetilde{X}}\right) = \prod_{i=1}^{n} p(X = X_i)$$

$$L(P.\lambda/\widetilde{X}) = \left((P + (1-P)e^{-\lambda}\right)^{Y} \prod_{i=1.X_{i\neq 0}}^{n} (1-P)e^{-\lambda} \frac{\lambda^{X_i}}{X_i!}$$

The logarithm function of likelihood function will be:

Set Eq. (11) and Eq. (12) equal to (0), it Will be give the following:

$$Ln(L) = Yln\left(P + (1-P)e^{-\lambda}\right) + (n-Y)ln(1-P) - (n-Y)\lambda + n\overline{X}ln(\lambda) - ln\left(\prod_{i=1}^{n} X_i!\right) \quad (10)$$

$$\frac{\partial ln(L)}{\partial \lambda} = \frac{-Y(1-P)e^{-\lambda}}{P + (1-P)e^{-\lambda}} - (n-Y) + \frac{n\overline{X}}{\lambda} \quad (11)$$

$$\frac{\partial ln(L)}{\partial P} = \frac{Y}{P + (1-P)e^{-\lambda}}\left(1 - e^{-\lambda}\right) - \frac{n-Y}{1-P} \quad (12)$$

:

$$\frac{n\overline{X}}{\hat{\lambda}_{MLE-ZIP}} = \frac{Y(1-\hat{P}_{MLE-ZIP})e^{-\hat{\lambda}_{MLE-ZIP}}}{\hat{P}_{MLE-ZIP} + (1-\hat{P}_{MLE-ZIP})e^{-\hat{\lambda}_{MLE-ZIP}}} + n - Y \quad (13)$$

$$\frac{Y\left(1 - e^{-\hat{\lambda}_{MLE-ZIP}}\right)Y\left(1-\hat{P}_{MLE-ZIP}\right)}{\hat{P}_{MLE-ZIP} + \left(1-\hat{P}_{MLE-ZIP}\right)e^{-\hat{\lambda}_{MLE-ZIP}}} = n - Y \quad (14)$$

The previous equations are nonlinear then Newton Raphson method is relied upon to obtain the maximum likelihood estimators.

4.2 Method of Moment Estimation (MME) for ZIP

The method involves determining estimators by equating the sample moments to the corresponding distribution parameters that need to be estimated [13,14]. This is achieved by calculating expectations and considering various degrees of the assumed distribution, which are functions dependent on its parameters. Sample moments will be

$$\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n} \quad and\ S^2 = \frac{\sum_{i=1}^{n}\left(x_i - \overline{X}\right)^2}{n-1}$$

$$\text{By using eq.}(4)\ \ \overline{X} = (1-P)\lambda$$

$$and\ \ P = 1 - \frac{\overline{X}}{\lambda}$$

$$By\ \ using\ eq.(5)\ \ S^2 = (1-P)\lambda(1+P\lambda) = \overline{X}(1+P\lambda) = \overline{X}(1+\left(\frac{\lambda-\overline{X}}{\lambda}\right)\lambda)$$

$$\frac{S^2}{\overline{X}} = 1 + \lambda - \overline{X}$$

$$\hat{\lambda}_{MME-ZIP} = \overline{X} + \frac{S^2}{\overline{X}} - 1 \tag{15}$$

$$\frac{S^2}{\overline{X}} = 1 + P\hat{\lambda}_{MME-ZIP} = 1 + P\left(\frac{S^2}{\overline{X}} + \overline{X} - 1\right)$$

$$\frac{S^2}{\overline{X}} - 1 = P\left(\frac{S^2}{\overline{X}} + \overline{X} - 1\right)$$

$$\hat{P}_{MME-ZIP} = \frac{S^2 - \overline{X}}{S^2 + (\overline{X})^2 - \overline{X}} \tag{16}$$

5. Simulation Experiments

In statistics, many researchers use simulation as an alternative system for real data in order to demonstrate the flexibility and scalability of distributions and others in processing [15,16,17]. Numerous simulation experiments were conducted to compare the two estimation methods for the parameters of the inflated Poisson regression model .These experiments were based on the initial parameters .Polluted number ($\lambda = 1,2,3$), inflation parameter($P = 0.1,0.3$),sample size ($n = 20,50,100$) ,(Iteration number In =1000). Estimation parameter of $\hat{\theta}_i = (\hat{\lambda}_{MME-ZIP}, \hat{P}_{MME-ZIP}, \hat{\lambda}_{MLE-ZIP}, \hat{P}_{MLE-ZIP})$ will be:

$$\hat{\theta}_i = \frac{1}{I_n}\sum_{j=1}^{I_n}\hat{\theta}_{ij} \tag{17}$$

Comparing estimators according to:

$$\zeta_i = Min\left(\left|\hat{\theta}_i - \theta_i\right|\right) \tag{18}$$

$$MSE_i = \frac{1}{I_n}\sum_{j=1}^{I_n}\left(\hat{\theta}_{ij} - \theta_i\right)^2 \tag{19}$$

Such that (ζ_i) represent Criteria of Minimum Absolute Different and (MSE_i) represent Mean Square Error Criteria.

6. Experimental Results

After carrying out the simulation experiments, we have the following results the 1st experiment with (P = 0.1), (λ = 1,2,3), (n = 20,50,100).

Table 1- the inflation parameter estimators, Minimum Absolute Different, best estimation method for ($P = 0.1$), ($\lambda = 1,2,3$), ($n = 20,50,100$)

λ	n	P_{mle}	P_{mom}	ζ_i	$Best$
1	20	0.198208	0.102681	0.002681	MOM
1	50	0.141866	0.150427	0.041866	MLE
1	100	0.103421	0.100749	0.000749	MOM
2	20	0.124914	0.121532	0.021532	MOM
2	50	0.108943	0.100351	0.000351	MOM
2	100	0.11717	0.130128	0.01717	MLE
3	20	0.173195	0.140144	0.040144	MOM
3	50	0.103764	0.100607	0.000607	MOM
3	100	0.109113	0.100281	0.000281	MOM

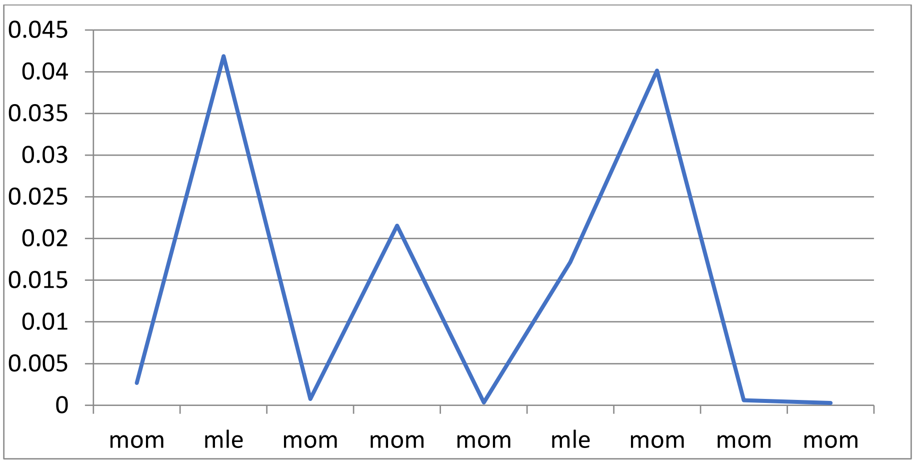

Figure -1 Minimum Absolute Different values for ($P = 0.1$), ($\lambda = 1,2,3$), ($n = 20,50,100$)

From table (1) and figure (1) we, can show that the best estimation method according to Minimum Absolute Different criteria for the previous simulation experiment with (P = 0.1), (λ = 1,2,3), (n = 20,50,100) for inflation estimator, the best estimation method was (mom) with (78%) success comparing with (mle)

Table 2- Mean Square Error with best estimation method for $(P = 0.1)$, $(\lambda = 1,2,3)$, $(n = 20,50,100)$

λ	n	MSE_{mle}	MSE_{mom}	$Best$
1	20	0.009934	0.000664	MOM
1	50	0.001985	0.010834	MLE
1	100	0.000484	0.00069	MLE
2	20	0.009539	0.000797	MOM
2	50	0.000321	0.000673	MLE
2	100	0.000623	0.000862	MLE
3	20	0.005274	0.000296	MOM
3	50	0.000642	0.000122	MOM
3	100	0.000708	0.000808	MLE

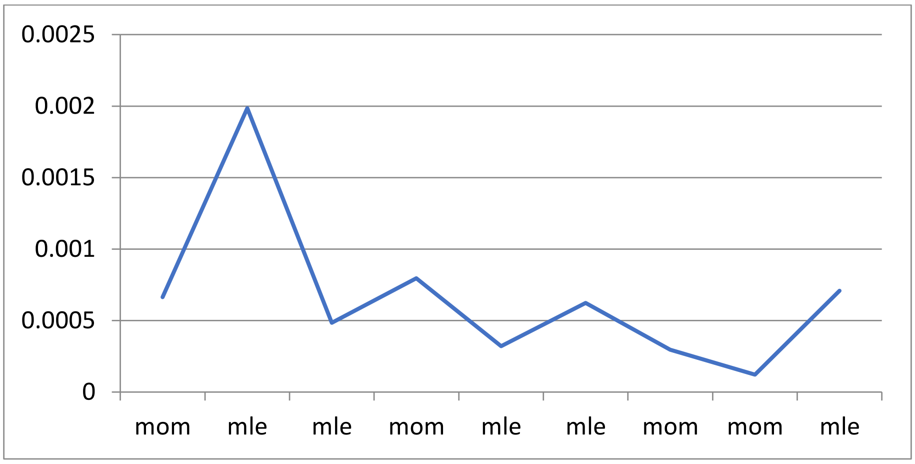

Figure -2 Mean Square Error values for $(P = 0.1)$, $(\lambda = 1,2,3)$, $(n = 20,50,100)$

From table (2) and figure (2), we can show that the best estimation method according to Mean square error criteria for the previous (9) simulation experiment with (P = 0.1), (λ = 1,2,3), (n = 20,50,100) for inflation estimator, the best estimation method was (mle) with (56%) success comparing with (mom).

Table 3- the inflation parameter estimators, Minimum Absolute Different, best estimation method for ($P = 0.3$), ($\lambda = 1,2,3$), ($n = 20,50,100$)

λ	n	P_{mle}	P_{mom}	ζ_i	$Best$
1	20	0.346076	0.29685	0.19685	MOM
1	50	0.302091	0.301177	0.201177	MOM
1	100	0.303252	0.30084	0.20084	MOM
2	20	0.446668	0.294896	0.194896	MOM
2	50	0.307123	0.311427	0.207123	MLE
2	100	0.331643	0.300595	0.200595	MOM
3	20	0.341056	0.310792	0.210792	MOM
3	50	0.291395	0.300399	0.191395	MLE
3	100	0.37758	0.300615	0.200615	MOM

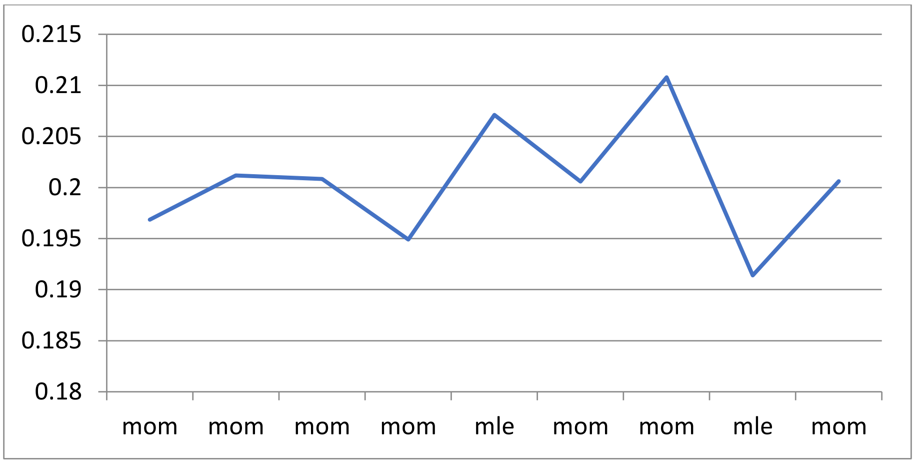

Figure -3 Minimum Absolute Different values for ($P = 0.3$), ($\lambda = 1,2,3$), ($n = 20,50,100$)

From table (3) and figure (3 we, can show that the best estimation method according to Minimum Absolute Different criteria for the previous (9) simulation experiment with ($P = 0.3$), ($\lambda = 1,2,3$), ($n = 20,50,100$) for inflation estimator, the best estimation method was (mom) with (78%) success comparing with (mle)

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Table 4- Mean Square Error with best estimation method for $(P = 0.3)$, $(\lambda = 1,2,3)$, $(n = 20,50,100)$

λ	n	MSE_{mle}	MSE_{mom}	$Best$
1	20	0.00298	0.000388	MOM
1	50	0.000829	0.000789	MOM
1	100	0.000807	5.41E-05	MOM
2	20	0.021674	0.000471	MOM
2	50	4.32E-05	0.00062	MLE
2	100	0.001169	0.000442	MOM
3	20	0.001944	0.000989	MOM
3	50	0.000567	0.000741	MLE
3	100	0.006131	0.000293	MOM

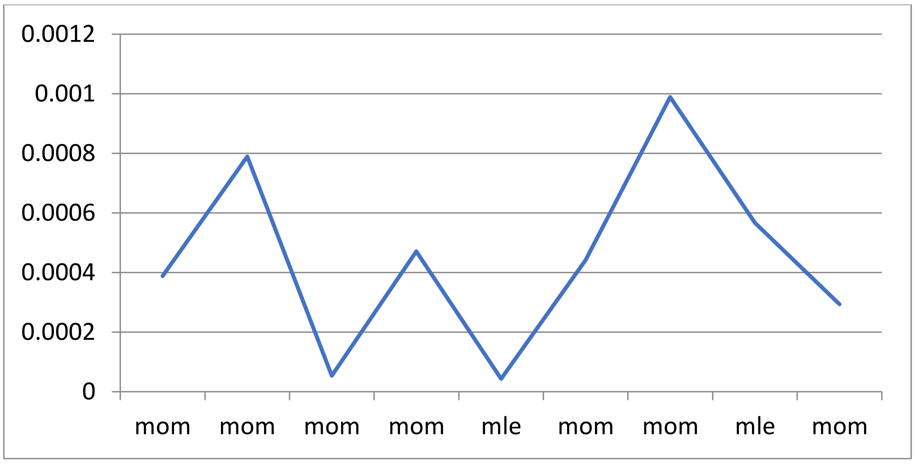

Figure -4 Mean Square Error values for $(P = 0.3)$, $(\lambda = 1,2,3)$, $(n = 20,50,100)$

From table (4) and figure (4) we, can show that the best estimation method according to Mean square error criteria for the previous simulation experiment with $(P = 0.3)$, $(\lambda = 1,2,3)$, $(n = 20,50,100)$ for inflation estimator, the best estimation method was (mom) with (78%) success comparing with (mle)

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Table 5- the Polluted estimators, Minimum Absolute Different, best estimation method for $(P = 0.1)$, $(\lambda = 1,2,3)$, $(n = 20,50,100)$

λ	n	λ_{mle}	λ_{mom}	ζ_i	λ
1	20	0.904359	1.001203	0.001203	MOM
1	50	1.115911	1.000486	0.000486	MOM
1	100	1.09792	1.20082	0.09792	MLE
2	20	1.864241	2.205929	0.135759	MLE
2	50	1.744037	2.000659	0.000659	MOM
2	100	1.983185	2.200904	0.016815	MLE
3	20	3.07278	2.399464	0.07278	MLE
3	50	3.271128	3.000653	0.000653	MOM
3	100	2.673267	3.000501	0.000501	MOM

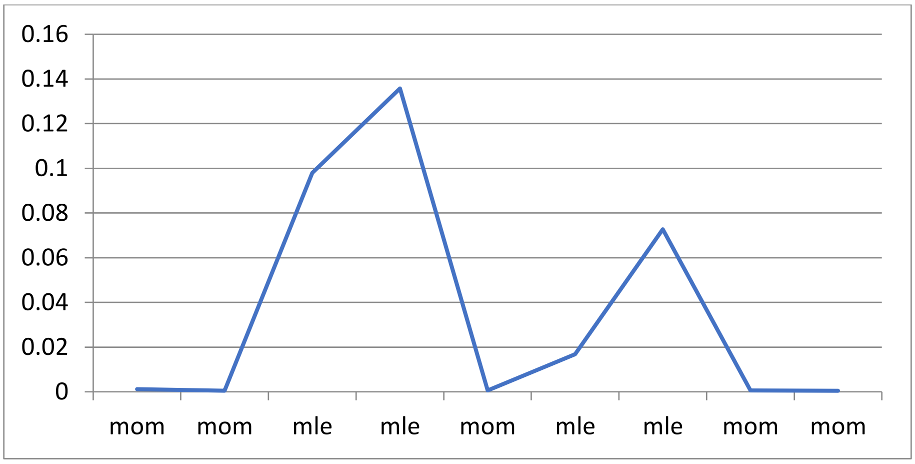

Figure -5 Minimum Absolute Different for Polluted estimators values for $(P = 0.1)$, $(\lambda = 1,2,3)$, $(n = 20,50,100)$

From table (5) and figure (5) we, can show that the best estimation method according to Minimum Absolute Different criteria for the previous simulation experiment with (P = 0.1), (λ = 1,2,3), (n = 20,50,100) for polluted estimator, the best estimation method was (mom) with (56%) success comparing with (mle)

Table 6- Mean Square Error for Polluted estimators with best estimation method for ($P = 0.1$), ($\lambda = 1,2,3$), ($n = 20,50,100$)

λ	n	MSE_{mle}	MSE_{mom}	$Best\ Meth$
1	20	0.009278	0.000387	MOM
1	50	0.013888	0.000864	MOM
1	100	0.009977	0.010029	MLE
2	20	0.018858	9.66E-05	MOM
2	50	0.065827	0.000645	MOM
2	100	0.000591	0.00028	MOM
3	20	0.006116	0.01027	MLE
3	50	0.073604	7.87E-05	MOM
3	100	0.107443	0.00063	MOM

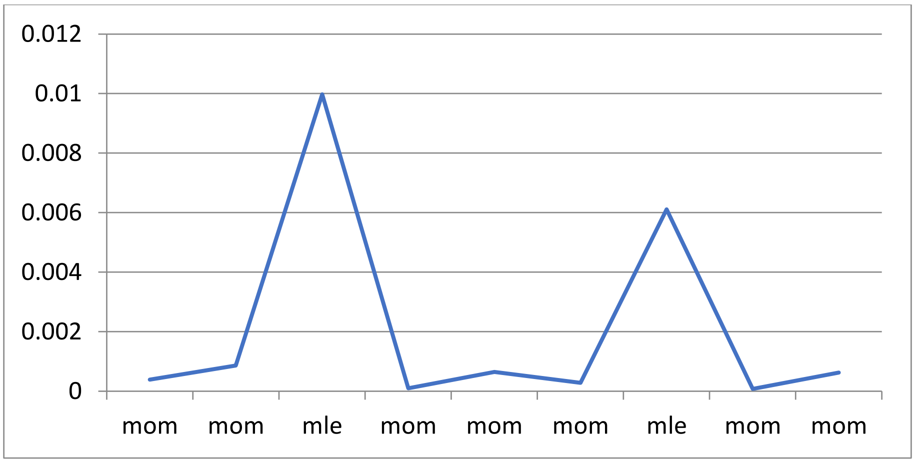

Figure -6 Mean Square Error values for Polluted estimators for ($P = 0.1$), ($\lambda = 1,2,3$), ($n = 20,50,100$)

From table (6) and figure (6) we, can show that the best estimation method according to Mean square error criteria for the previous simulation experiment with ($P = 0.1$), ($\lambda = 1,2,3$), ($n = 20,50,100$) for polluted estimator, the best estimation method was (mom) with (78%) success comparing with (mle).

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Table 7- The Polluted estimators, Minimum Absolute Different, best estimation method for ($P = 0.3$), ($\lambda = 1,2,3$), ($n = 20,50,100$)

λ	n	λ_{mle}	λ_{mom}	ζ_i	λ
1	20	1.189616	0.993918	0.006082	MOM
1	50	0.983669	1.000784	0.000784	MOM
1	100	0.800062	1.200197	0.199938	MLE
2	20	1.716925	1.993862	0.006138	MOM
2	50	1.740712	2.300238	0.259288	MLE
2	100	2.151285	2.000281	0.000281	MOM
3	20	3.171608	3.402468	0.171608	MLE
3	50	3.043564	3.001276	0.001276	MOM
3	100	3.123115	3.000789	0.000789	MOM

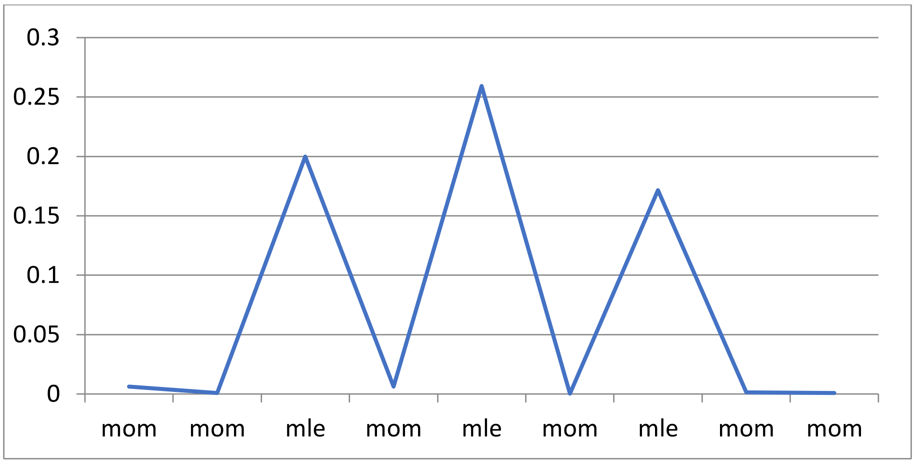

Figure -7 Minimum Absolute Different for Polluted estimators values for ($P = 0.3$), ($\lambda = 1,2,3$), ($n = 20,50,100$)

From table (7) and figure (7) we, can show that the best estimation method according to Minimum Absolute Different criteria for the previous simulation experiment with (P = 0.3), (λ = 1,2,3), (n = 20,50,100) for polluted estimator, the best estimation method was (mom) with (67%) success comparing with (mle).

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Table 8- Mean Square Error for Polluted estimators with best estimation method for ($P = 0.3$), ($\lambda = 1,2,3$), ($n = 20,50,100$)

λ	n	MSE_{mle}	MSE_{mom}	$Best\ Meth$
1	20	0.036085	0.000113	MOM
1	50	0.000523	0.000774	MLE
1	100	0.041085	0.000213	MOM
2	20	0.080533	0.000327	MOM
2	50	0.06807	0.000511	MOM
2	100	0.023235	0.000902	MOM
3	20	0.451352	0.000409	MOM
3	50	0.002137	0.000887	MOM
3	100	0.01566	0.000569	MOM

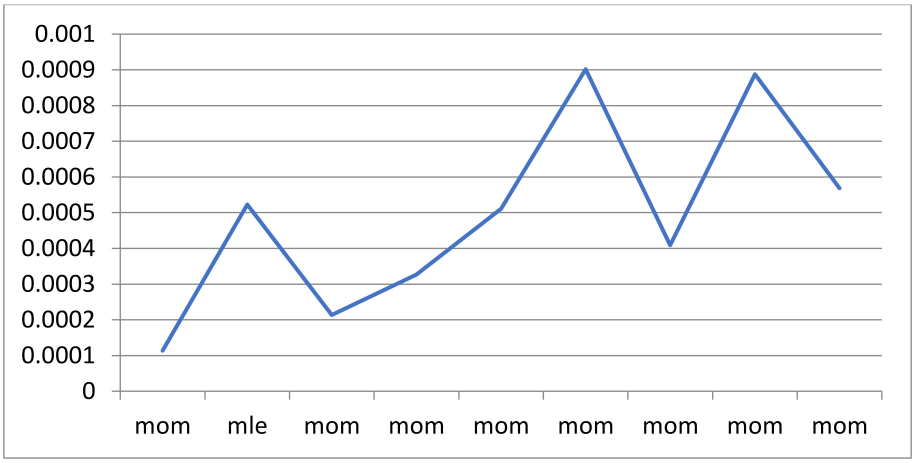

Figure -8 Mean Square Error values for Polluted estimators for ($P = 0.3$), ($\lambda = 1,2,3$), ($n = 20,50,100$)

From table (8) and figure (8) we, can show that the best estimation method according to Mean square error criteria for the previous simulation experiment with ($P = 0.3$), ($\lambda = 1,2,3$), ($n = 20,50,100$) for polluted estimator, the best estimation method was (mom) with (89%) success comparing with (mle).

7. Conclusions and Suggestions

Upon analysing the outcomes of the simulation experiments, it became evident to us that the (mom) method outperformed the (mle) method under varying experimental conditions. The estimations of (polluted estimators) were influenced by the diverse conditions of the simulation experiments, while the estimations of (inflation estimators) were also influenced by these conditions. To estimate the pollution parameters of the amplified zero-Poisson regression, Bayesian and white noise estimators can be employed.

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