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New Formulation for Linear and Quadratic Production Models

Rasheed Al-Salih [1]*, Watheq Laith [2], Abdelmonaem Jornaz [3]

[1] Department of Mathematics, College of Education, University of Sumer, Thi qar, Iraq
[2] Department of Mechanical Engineering, College of Engineering, University of Sumer, Thi qar, Iraq
[3] Department of Mathematics, College of Science, Northwest Missouri State University, Maryville, MO, USA
* Email address of the Corresponding Author: basheerreh79@gmail.com

Abstract

In this paper, we present a new formulation for some applications of the dynamic mathematical programming problems using time scale. These applications include linear and quadratic objective function. Also, this formulation is an extension for the existing one. In addition, the continuous models and quantum models are obtained by choosing T = R and $T = qN^0$ receptively. Moreover, the primal and the dual of the dynamic mathematical programming models on time scales are constructed for the linear and quadratic production model. This concepts of the duality theory will help the decision maker to choose either the primal model or the dual model for finding the optimal solution in particular for isolated time scales set. Furthermore, using this formulation the optimal solution can be obtained easily using some well-known discrete optimization methods such us simplex method and interior point method.

Keywords: Dynamic production model, quantum calculus, primal model, dual model, time scales.

صياغة جديدة لنماذج الانتاج الخطية والتربيعية

ا.د.رشيد بشير رحيمة [1]*, ا.د. واثق حياوي لايذ[2], ا.م.د. عبد المنعم جرناز

[1]قسم الرياضيات, كلية التربية, جامعة سومر, ذي قار, العراق
[2]قسم الهندسة الميكانيكية, كلية الهندسة, جامعة سومر, ذي قار, العراق
[3]قسم الرياضيات, جامعة ولاية شمال غرب ميزوري , ميزوري, الولايات المتحدة الامريكية

الخلاصة

في هذا البحث تم عرض صياغة جديدة لبعض تطبيقات البرمجة الرياضية الديناميكية باستخدام مقياس الزمن. هذه التطبيقات تشمل دالة الهدف الخطية والتربيعية. وكذلك تعتبر هذه الصياغة توسيع للصياغة الموجودة حاليا. بالإضافة الى ان النماذج المستمرة والكمية يتم الحصول عليها بسهولة من الصياغة الجديدة وذلك بفرض المجموعة تساوي الاعداد الحقيقية او الكمية. ايضا يتم صياغة النموذج الاولي والثنائي مع دالة الهدف الخطية والتربيعية. ان مفهوم النظرية الثنائية سوف يساعد متخذ القرار من اختيار النموذج الاولي او الثنائي لإيجاد الحل الامثل للمشكلة وباستخدام مجموعة معينة ذات قيم معزولة.

الكلمات المفتاحية: نماذج الانتاج الديناميكية، الحسبان الكمي، الانموذج الاولي، الانموذج المقابل، مقياس الزمن.

1. Introduction

It is well known that a dynamical model presents a mathematical model that describes how this model behave over time. The dynamical models have recently used to characterize of various phenomena in fields for instances physics, engineering, biology, economics, and more related area. Also, dynamical models play an important role in understanding the dynamics of complex systems and predicting their future states. In addition, there are two types of dynamical model which are the discrete and continuous models. These models have been studied from different perspectives such as solutions methods, algorithms and real world applications.

On the other hand, the concept of time scales was first introduced by Stefan Hilger in the early 1980s, and it has received significant attention in various scientific aspects. The time scales framework provides a more comprehensive understanding of dynamic processes by allowing researchers to bridge the gap between continuous and discrete models. Likewise, dynamic mathematical programming models have verity of real world applications for example in physics, in engineering, business and finance.

The dynamic continuous time models have studied from different perspectives, for more details we refer to [9, 10]. On the other hand, new formulation of LP models on time scales has presented by [1, 4] using time scales. FL models have been established using time scales by [2]. Non-linear programming problems have been introduced using time scales by [3]. Some applications of mathematical programming are established using quantum calculus analogues as in [5, 6, 8].

The objective of this paper is to extend the formulation that is presented in [5, 6] to general class of time scales for which the quantum calculus and the continuous model are obtains as special cases and the optimal solution can be obtained easily.

2. Discrete Mathematical Programming Models Setup

Discrete mathematical models consist of objective function and constraints and the goal is to find the optimal solution that provides the maximum profit or minimum cost. The mathematical primal model of discrete LP model is described as follows:

$$Max\ z = c^T x \tag{1}$$

S.t. $Ax \leq b$

$x \geq 0.$

Where,

$$c = [c_j]_n \ , x = [x_j]_n \ , A = [a_{ij}]_{m*n} \ , b = [b_i]_m,$$

$c_j, d_j, x_j, a_{ij} \in R.$

The dual linear model as follows.

$$Min\ W = b^T y \tag{2}$$

S.t. $A^T y \geq c$

$y \geq 0.$

Where,

$$c = [c_j]_n \ ,y = [y_j]_m \ , A = [a_{ij}]_{m*n} \ , b = [b_i]_m,$$

$c_j, , y_j, a_{ij} \in R.$

The primal and dual discrete programming model have many applications see for examples

[11],[12],[13] and these models can be solved using some well-known methods such us graphical method, simplex method, interior point method, and revised simplex method.

3. Overview of Time Scales

Throughout, “T is the time scale, σ is the forward jump operator, μ is the graininess, f : T → R is a function, f^{Δ} is the delta derivative of f, and $\int_a^b f(t)\Delta t$ is the time scales integral of f between $a,b \in$ T” for more details about the basic concepts of time scales we refer to [1,7].

In this paper, we will use the following time scales integrals formulas as presented in [1,7].

1- $If\ T = Z,$ then

$$\int_a^b f(t)\Delta t = \sum_{k=a}^{b-1} f(k)\, , where\ a, b \in T\ with\ a < b.$$

2- $Let\ h > 0,\ If\ T = hZ = \{hk : k \in Z\}\, , then$

$$\int_a^b f(t)\Delta t = h \sum_{k=\frac{a}{h}}^{\frac{b}{h}-1} f(kh)\, , where\ a, b \in T\ with\ a < b.$$

3- $If\ T = q^{N_0},$ then

$$\int_{q^m}^{q^n} f(t)\Delta t = (q-1) \sum_{k=m}^{n-1} q^k f(q^k)\, , where\ m, n \in N_0\ with\ m < n.$$

4- $If\ T = R,$ then

$$\int_a^b f(t)\Delta t = \int_a^b f(t)dt, \qquad where\ a, b \in T\ with\ a < b.$$

4. Continuous-time Leontief Production Model

Leontief production model is a well know economics model. There are two types of this economics model open model and closed model. For more details, we refer to [5,6,9]. The primal and dual models for linear model are presented as in [5,6,9] respectively in Table 1:

Table 1- Primal and dual formulation of Leontief production model

PLLPMTS	DLLPMTS
$Max\ W(x) = \int_0^T (\acute{a}(t)x(t))dt$ $s.t.\ A(t)x(t) \le c(t) + \int_0^t Mx(s)ds\,,$ $and\ x \in E_n, \quad x(t) \ge 0, t \in [0\,.T]$	$Min\ G(y) = \int_0^T (c(t)y(t))dt$ $s.t.\ \acute{A}y(t) \ge a(t) + \int_t^T \acute{M}y(s)ds\,,$ $and\ y \in E_m, \quad y(t) \ge 0, \quad t \in [0\,.T$

Now, the primal and dual models for quadratic model are presented as in [5,6,9] respectively in Table 2.

Table 2- Primal and dual formulation of quadratic Leontief production

PQLPMTS	DQLPMTS
$Max\ W(x) = \int_0^T \left(\acute{a}(t)x(t) + \frac{1}{2}\acute{x}(t)Dx(t)\right) dt$ $s.t.\ A(t)x(t) \le c(t) + \int_0^t Mx(s)ds\,,$ $and\ x \in E_n, \quad x(t) \ge 0, t \in [0\,.T]$	$Min\ G(y) = \int_0^T (-\frac{1}{2}\acute{v}(t)Dv(t) + c(t)y(t))$ $s.t.\ \acute{A}y(t) \ge a(t) + \int_t^T \acute{M}y(s)ds\,,$ $and\ y \in E_m, \quad y(t) \ge 0,\ t \in [0\,.T]$

5. Quadratic Leontief models on Time Scales

The authors in [5,6] have been formulated the linear and quadratic Leontief production models in quantum calculus. In this Section, we extend this formulation to general time scales. Throughout this paper, as in [1,5,6] we use "J to denote the time scale interval J = [0,*T*] ∩T,

and by E_k, we denote the space of all rd-continuous functions from J into R*k*".
The primal and the dual linear Leontief production model on time scales are respectively in table 3 as:

Table 3- Time scale formulation of linear Leontief production model

PLLPMTS	DLLPMTS
$Max\ W(x) = \int_0^{\sigma(T)} (\acute{a}(t)x(t))\Delta t$ $s.t.\ A(t)x(t) \le c(t) + \int_0^t Mx(s)\Delta s\,, \quad q^n \in$ $and\ x \in E_n, \quad x(t) \ge 0, t \in J\,.$	$Min\ G(y) = \int_0^{\sigma(T)} (c(t)y(t))\Delta t$ $s.t.\ \acute{A}y(t) \ge a(t) + \int_{\sigma(t)}^{\sigma(T)} \acute{M}y(s)\Delta s\,,$ $and\ y \in E_m, \quad y(t) \ge 0, \quad t \in$

The primal and dual quadratic Leontief production model on time scales are respectively in Table 4 as:

Table 4- Time scale formulation of quadratic Leontief production model

PQLPMTS	DQLPMTS
$Max\ W(x) = \int_0^{\sigma(T)} \left(\acute{a}(t)x(t) + \frac{1}{2}\acute{x}(t)Dx(t)\right)\Delta t$	$Min\ G(y) = \int_0^{\sigma(T)} \left(-\frac{1}{2}\acute{v}(t)Dv(t) + c(t)y(t)\right.$
$s.t.\ A(t)x(t) \le c(t) + \int_0^t Mx(s)\Delta s,\quad q^n \in J$	$s.t.\ \acute{A}y(t) \ge a(t) + \int_{\sigma(t)}^{\sigma(T)} \acute{M}y(s)\Delta s,\quad q^n \in$
$and\ x \in E_n,\quad x(t) \ge 0, t \in J.$	$and\ y \in E_m,\quad y(t) \ge 0,\qquad t \in J.$

6. Examples

6.1 Example 1

“ $= Z$", then

$$\sigma(t) = t + 1, \qquad \mu(t) \equiv 1, \qquad \text{for} \qquad t \in T,$$

and the time scales production linear and quadratic models will become discrete models respectively as follows.

Case1: linear model in Table 5 as follows.

Table 5- Linear production model on $T = Z$

PLLPMTS	DLLPMTS
$Max\ W(x) = \int_0^{\sigma(T)} \left(\acute{a}(t)x(t)\right)\Delta t$	$Min\ G(y) = \int_0^{\sigma(T)} \left(c(t)y(t)\right)\Delta t$
$Max\ W(x) = \sum_{k=a}^{b-1} \left(\acute{a}(k)x(k)\right),$	$Min\ G(y) = \sum_{k=a}^{b-1} \left(c(k)y(k)\right)$
$where\ a, b \in T\ with\ a < b$	$s.t. \acute{A}y(t) \ge a(t) + \int_{\sigma(t)}^{\sigma(T)} \acute{M}y(s)\Delta s,\quad t \in J$
$s.t.\ A(t)x(t) \le c(t) + \int_0^t Mx(s)\Delta s,\quad t \in J$	$s.t.\ \acute{A}y(t) \ge a(t) + \sum_{k=a}^{b-1} \acute{M}y(k)$
$s.t.\ A(t)x(t) \le c(t) + \sum_{k=a}^{b-1} Mx(k)$	$where\ a, b \in T\ with\ a < b$
$and\ x \in E_n,\quad x(t) \ge 0, t \in Z.$	$and\ y \in E_m,\quad y(t) \ge 0,\qquad t \in Z.$

Case2: quadratic model in Table 6 as follows.

Table 6- Linear production model on $T = Z$

PQLPMTS	DQLPMTS

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$Max\ W(x) = \int_0^{\sigma(T)} \left(\acute{a}(t)x(t) + \frac{1}{2}\acute{x}(t)Dx(t) \right) \Delta t$	$Min\ G(y) = \int_0^{\sigma(T)} \left(-\frac{1}{2}\acute{v}(t)Dv(t) + c(t)y(t) \right.$
$Max\ W(x) = \sum_{k=a}^{b-1} \left(\acute{a}(k)x(k) + \frac{1}{2}\acute{x}(k)Dx(k) \right)$	$Min\ G(y) = \sum_{k=a}^{b-1} \left(-\frac{1}{2}\acute{v}(k)Dv(k) + c(k)y(k \right.$
$s.t.\ A(t)x(t) \le c(t) + \sum_{k=a}^{b-1} Mx(k)$	$s.t.\ \acute{A}y(t) \ge a(t) + \int_{\sigma(t)}^{\sigma(T)} \acute{M}y(s)\Delta s\ , \quad t \in$
$and\ x \in\ E_n, \quad x(t) \ge 0, t \in Z\ .$ $where\ a, b \in T\ with\ a < b$	$s.t.\ \acute{A}y(t) \ge a(t) + \sum_{k=a}^{b-1} Mx(k)$ $and\ y \in\ E_m, \quad y(t) \ge 0, \qquad t \in Z\ .$ $where\ a, b \in T\ with\ a < b$

6.2 Example 2

$" T = hZ = \{hk : k \in Z, h > 0\} ", then$

$\Sigma(t) = t + h, \quad \mu(t) \equiv h, \qquad$ for $\qquad t \in T,$

and the time scales production linear and quadratic models will become discrete models respectively as follows.

Case1: linear model in Table 7 as follows.

Table 7- Linear production model on $T = Z$

PLLPMTS	DLLPMTS
$Max\ W(x) = \int_0^{\sigma(T)} \left(\acute{a}(t)x(t) \right) \Delta t$	$Min\ G(y) = \int_0^{\sigma(T)} \left(c(t)y(t) \right) \Delta t$
$Max\ W(x) = h \sum_{k=\frac{a}{h}}^{\frac{b}{h}-1} \left(\acute{a}(kh)x(kh) \right),$	$Min\ G(y) = h \sum_{k=\frac{a}{h}}^{\frac{b}{h}-1} \left(c(kh)y(kh) \right)$
$s.t.\ A(t)x(t) \le c(t) + \int_0^t Mx(s)\Delta s\ , \quad t \in J$	$s.t. \acute{A}y(t) \ge a(t) + \int_{\sigma(t)}^{\sigma(T)} \acute{M}y(s)\Delta s\ , \quad t \in J$
$s.t.\ A(t)x(t) \le c(t) + h \sum_{k=\frac{a}{h}}^{\frac{b}{h}-1} Mx(kh)$	$s.t.\ \acute{A}y(t) \ge a(t) + h \sum_{k=\frac{a}{h}}^{\frac{b}{h}-1} \acute{M}y(kh)$
$where\ a, b \in T\ with\ a < b$ $and\ x \in\ E_n, \quad x(t) \ge 0, t \in hZ\ .$	$where\ a, b \in T\ with\ a < b$ $and\ y \in\ E_m, \quad y(t) \ge 0, \qquad t \in hZ\ .$

Case2: quadratic model in Table 8 as follows.

Table 8- Linear production model on $T = Z$

PQLPMTS	DQLPMTS
$Max\ W(x) = \int_0^{\sigma(T)} \left(\acute{a}(t)x(t) + \frac{1}{2}\acute{x}(t)Dx(t)\right)\Delta t$	$Min\ G(y) = \int_0^{\sigma(T)} \left(-\frac{1}{2}\acute{v}(t)Dv(t) + c(t)y(t)\right.$
$= h\sum_{k=\frac{a}{h}}^{\frac{b}{h}-1}\left(\acute{a}(kh)x(kh) + \frac{1}{2}\acute{x}(kh)Dx(kh)\right)$	$= h\sum_{k=\frac{a}{h}}^{\frac{b}{h}-1}\left(-\frac{1}{2}\acute{v}(kh)Dv(kh) + c(kh)y(kh)\right)$
$s.t.\ A(t)x(t) \le c(t) + h\sum_{k=\frac{a}{h}}^{\frac{b}{h}-1} Mx(kh)$	$s.t.\ \acute{A}y(t) \ge a(t) + \int_{\sigma(t)}^{\sigma(T)} \acute{M}y(s)\Delta s\ ,\ \ t \in$
	$s.t.\ \acute{A}y(t) \ge a(t) + h\sum_{k=\frac{a}{h}}^{\frac{b}{h}-1} Mx(kh)$
$and\ x \in E_n,\ \ x(t) \ge 0, t \in Z\ .$ $where\ a, b \in T\ with\ a < b$	$and\ y \in E_m,\ \ y(t) \ge 0,\ \ \ t \in Z\ .$ $where\ a, b \in T\ with\ a < b$

7. Remarks

1-. " $T = R$", then

$$\sigma(t) = t, \qquad \mu(t) \equiv 0, \qquad \text{for} \qquad t \in T,$$

and the time scales production linear and quadratic models will become continuous-time models respectively as in [9].

2- Let " > 1. If $T = q^{N_0} = \{q^n : n \in N_0\}$", then

$$\sigma(t) = qt, \qquad \mu(t) = (q-1)t, \text{ for } \qquad t \in T,$$

and the time scales production linear and quadratic models will become quantum models respectively as in [5],[6].

8. Conclusions

A formulation for some applications of mathematical programming problems using time scales technique has been presented. This formulation is an extension of quantum calculus analogues that are presented by [5],[6]. Based on our new formulation we can obtain the dynamic continuous time model and dynamic quantum calculus model by setting T = R and T = q^{N0} receptively. Moreover, we can obtain the optimal solution if we restrict the new formulation on isolated time scales sets such as T = q^{N0} and T = Z. In addition, the dual model for these applications is established for linear and quadratic models which is very important role in optimization theory. Also, the dual model will provide alternative way to formulate the problem and provides bounds on the optimal solution and may offer a different perspective on the problem. This work can extend to other dynamic models such as in engineering, physics and other related applications.

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