



## Applications of the PG (3,7) in Coding Theory

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### Abstract

What we are doing in this research is a description of the applications of the PG(3,q) projective space on the 7th order Galois field in the projective code [n,k,d]<sub>q</sub> such that the length of the code parameters is n, and the dimension of the code k and the minimum distance are calculated. d with error correction e according to the incidence matrix. This research also provides examples and theories of the links between combinatorial structures and coding theory. The research method relies on constructing points, lines, and levels in PG(3,q). Using generated matrices

**Keywords:** Finite projective space, incidence matrix, linear code , good code , perfect code.

### تطبيقات الفضاء الإسقاطي PG(3,7) في نظرية الترميز

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### الخلاصة

ان ما نقوم به في هذا البحث هو وصف لتطبيقات الفضاء الإسقاطي PG(3,q) على مجال جالوا من الرتبة 7 في الكود الإسقاطي [n,k,d]<sub>q</sub> بحيث يكون طول معلمات الكود n ، و تم حساب البعد للرمز k والمسافة الدنيا d مع تصحيح الخطأ e وفق مصفوفة الوقوع. كما يقدم هذا البحث أمثلة ونظريات للروابط بين الهياكل التوافقية ونظرية التشفير. ويعتمد أسلوب البحث على بناء النقطة والخطوط والمستويات في PG(3,q). باستخدام المصفوفات المولدة

الكلمات المفتاحية: الفضاء الإسقاطي المحدود، مصفوفة الحدوث، الكود الخطي، الكود الجيد , الكود المثالي .

### 1. Introduction

When mentioning the coding theory, one must look at what many mathematicians have done, specifically studying the application of projective space to the Kahlua field, where many theories and definitions have emerged about the relationships between finite projective geometry and the coding theory. For example, Hirschfeld [1] showed some theories and definitions about the relationships between finite projective geometry and coding theory. Finite projective geometry and coding theory. Al-Saraji [2, 3] also translated the links between the



projective level of order 17 and error correction codes and presented some other important results.

Hill [4] presented the concepts and tools of coding theory. Al-Zangana [5, 6, 7, 8] discussed the relationship between the projective level of order 19 and error-correcting codes. Now we look at the projective space on a finite field of order  $q$ , where  $q=7$

## 2. Construction of PG(3,7)

The polynomial  $g(x) = x^4 + T^5x^3 + T^3x^2 + Tx + T^4$  is primitive over  $F_7 = [0,1, T, T^2, T^3, T^4, T^5]$ , where  $T=5$  is a primitive element of  $F_7$ , since  $g(0) = T^4, g(1) = T^5, g(T) = 1, g(T^2) = T^3, g(T^3) = 1, g(T^4) = T^3$  and  $g(T^5) = T^4$

**Theorem 2.1** : [6] sphere packing or Hamming bound

A  $q$ -ary  $(n, M, 2e+1)$  – code  $C$  satisfies

$$M \left\{ \binom{n}{0} + \binom{n}{1}(q-1) + \dots + \binom{n}{e}(q-1)^e \right\} \leq q^n$$

**Corollary 2.2** [2]: A  $q$ -ary  $(n, M, 2e+1)$  code  $C$  is perfect if and only if equality holds in Theorem 2.1

**Definition 2.3** [5]: A  $q$ -ary code  $C$  of length  $n$  is a subset of  $(F_q)$

**Definition 2.4 Linear Codes** [9] :

The minimum distance  $d$  of a non-trivial code  $C$  is given by

$$d = \min \{ d(x, y) | x \in C, y \in C, x \neq y \}$$

$$P_i = [1, 0, 0, 0] \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ T^2 & 1 & T^4 & T \end{pmatrix}^i, i = 0, 1, 2, \dots, 400,$$

$$l_i = l_1 \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ T^2 & 1 & T^4 & T \end{pmatrix}^i, i = 1, 2, \dots, 400,$$

T

$$plane_i = plane_1 \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ T^2 & 1 & T^4 & T \end{pmatrix}^i, i = 1, 2, \dots, 400,$$

Table (1) The points of  $P(3,7)$  are:



<b>p1</b>	(1, 0, 0, 0)	<b>p135</b>	(T4, T2, T3, 1)	<b>p269</b>	(T5, T, 1, 0)
<b>p2</b>	(0, 1, 0, 0)	<b>p136</b>	(1, T3, T, 1)	<b>p270</b>	(0, T5, T, 1)
<b>p3</b>	(0, 0, 1, 0)	<b>p137</b>	(T3, T5, T, 1)	<b>p271</b>	(T3, T, T2, 1)
<b>p4</b>	(0, 0, 0, 1)	<b>p138</b>	(T3, 0, T2, 1)	<b>p272</b>	(T4, 0, 0, 1)
<b>p5</b>	(T, T5, T3, 1)	<b>p139</b>	(T4, 0, 1, 1)	<b>p273</b>	(T, T4, T3, 1)
<b>p6</b>	(1, T, T5, 1)	<b>p140</b>	(T5, T2, T, 1)	<b>p274</b>	(1, T, 1, 1)
<b>p7</b>	(T2, T4, 0, 1)	<b>p141</b>	(T3, T3, T4, 1)	<b>p275</b>	(T5, T, 0, 1)
<b>p8</b>	(T, 1, T, 1)	<b>p142</b>	(T2, 0, 1, 0)	<b>p276</b>	(T, T, 0, 1)
<b>p9</b>	(T3, T4, 1, 1)	<b>p143</b>	(0, T2, 0, 1)	<b>p277</b>	(T, T2, 0, 1)
<b>p10</b>	(T5, 0, T5, 1)	<b>p144</b>	(T, T5, T2, 1)	<b>p278</b>	(T, T2, T2, 1)
<b>p11</b>	(T2, T2, T4, 1)	<b>p145</b>	(T4, T5, T3, 1)	<b>p279</b>	(T4, T5, T5, 1)
<b>p12</b>	(T5, T4, 1, 0)	<b>p146</b>	(1, T3, T5, 1)	<b>p280</b>	(T2, T5, T, 1)
<b>p13</b>	(0, T5, T4, 1)	<b>p147</b>	(T2, T4, 1, 1)	<b>p281</b>	(T3, T2, T2, 1)
<b>p14</b>	(T, T5, 1, 0)	<b>p148</b>	(T5, T4, T5, 1)	<b>p282</b>	(T4, 0, T5, 1)
<b>p15</b>	(0, T, T5, 1)	<b>p149</b>	(T2, T2, T2, 1)	<b>p283</b>	(T2, T5, T4, 1)
<b>p16</b>	(T2, 1, 0, 1)	<b>p150</b>	(T4, T3, T5, 1)	<b>p284</b>	(T, 1, 1, 0)
<b>p17</b>	(T, 1, T4, 1)	<b>p151</b>	(T2, T5, 1, 1)	<b>p285</b>	(0, T, 1, 1)
<b>p18</b>	(T3, T4, 1, 0)	<b>p152</b>	(T5, T4, T4, 1)	<b>p286</b>	(T5, T3, 0, 1)
<b>p19</b>	(0, T3, T4, 1)	<b>p153</b>	(1, 1, 1, 0)	<b>p287</b>	(T, T, T5, 1)
<b>p20</b>	(T2, 1, 1, 0)	<b>p154</b>	(0, 1, 1, 1)	<b>p288</b>	(T2, T3, 0, 1)
<b>p21</b>	(0, T2, 1, 1)	<b>p155</b>	(T5, T3, T2, 1)	<b>p289</b>	(T, 1, T5, 1)
<b>p22</b>	(T5, T3, 1, 1)	<b>p156</b>	(T4, T4, T2, 1)	<b>p290</b>	(T2, T3, T5, 1)
<b>p23</b>	(T5, T5, T3, 1)	<b>p157</b>	(T4, T, T4, 1)	<b>p291</b>	(T2, T, 1, 1)
<b>p24</b>	(1, 1, T5, 1)	<b>p158</b>	(T3, 1, 0, 0)	<b>p292</b>	(T5, T4, 0, 1)
<b>p25</b>	(T2, T4, T5, 1)	<b>p159</b>	(0, T3, 1, 0)	<b>p293</b>	(T, T, T, 1)
<b>p26</b>	(T2, T, T2, 1)	<b>p160</b>	(0, 0, T3, 1)	<b>p294</b>	(T3, T4, 0, 1)
<b>p27</b>	(T4, T3, 0, 1)	<b>p161</b>	(1, T4, T2, 1)	<b>p295</b>	(T, 0, T, 1)
<b>p28</b>	(T, T4, T5, 1)	<b>p162</b>	(T4, 1, T4, 1)	<b>p296</b>	(T3, T4, T5, 1)
<b>p29</b>	(T2, T3, T2, 1)	<b>p163</b>	(T3, 1, 1, 0)	<b>p297</b>	(T2, 0, T2, 1)
<b>p30</b>	(T4, T3, T2, 1)	<b>p164</b>	(0, T3, 1, 1)	<b>p298</b>	(T4, T3, 1, 1)
<b>p31</b>	(T4, T, T2, 1)	<b>p165</b>	(T5, T3, T3, 1)	<b>p299</b>	(T5, T2, T3, 1)
<b>p32</b>	(T4, T, 0, 1)	<b>p166</b>	(1, 1, T4, 1)	<b>p300</b>	(1, 1, T, 1)
<b>p33</b>	(T, T4, 0, 1)	<b>p167</b>	(T3, T5, 1, 0)	<b>p301</b>	(T3, T5, 1, 1)
<b>p34</b>	(T, T2, T, 1)	<b>p168</b>	(0, T3, T5, 1)	<b>p302</b>	(T5, 0, T4, 1)
<b>p35</b>	(T3, T4, T4, 1)	<b>p169</b>	(T2, 1, 1, 1)	<b>p303</b>	(T4, T4, 1, 0)
<b>p36</b>	(1, 0, 1, 0)	<b>p170</b>	(T5, T4, T2, 1)	<b>p304</b>	(0, T4, T4, 1)
<b>p37</b>	(0, 1, 0, 1)	<b>p171</b>	(T4, T4, T4, 1)	<b>p305</b>	(1, T4, 1, 0)
<b>p38</b>	(T, T5, T4, 1)	<b>p172</b>	(1, T3, 1, 0)	<b>p306</b>	(0, 1, T4, 1)
<b>p39</b>	(T, T2, 1, 0)	<b>p173</b>	(0, 1, T3, 1)	<b>p307</b>	(T3, T, 1, 0)
<b>p40</b>	(0, T, T2, 1)	<b>p174</b>	(1, T4, T3, 1)	<b>p308</b>	(0, T3, T, 1)
<b>p41</b>	(T4, T2, 0, 1)	<b>p175</b>	(1, T2, 1, 1)	<b>p309</b>	(T3, T, T, 1)
<b>p42</b>	(T, T4, T2, 1)	<b>p176</b>	(T5, T, 1, 1)	<b>p310</b>	(T3, 0, 0, 1)
<b>p43</b>	(T4, T5, T4, 1)	<b>p177</b>	(T5, T5, 0, 1)	<b>p311</b>	(T, 0, T3, 1)
<b>p44</b>	(T, T4, 1, 0)	<b>p178</b>	(T, T, 1, 1)	<b>p312</b>	(1, T, T2, 1)
<b>p45</b>	(0, T, T4, 1)	<b>p179</b>	(T5, 1, 0, 1)	<b>p313</b>	(T4, 1, 0, 1)
<b>p46</b>	(T2, 1, 0, 0)	<b>p180</b>	(T, T, T4, 1)	<b>p314</b>	(T, T4, T4, 1)
<b>p47</b>	(0, T2, 1, 0)	<b>p181</b>	(T5, 1, 0, 0)	<b>p315</b>	(1, T, 1, 0)
<b>p48</b>	(0, 0, T2, 1)	<b>p182</b>	(0, T5, 1, 0)	<b>p316</b>	(0, 1, T, 1)
<b>p49</b>	(T4, T2, 1, 1)	<b>p183</b>	(0, 0, T5, 1)	<b>p317</b>	(T3, T, 1, 1)
<b>p50</b>	(T5, T2, 1, 1)	<b>p184</b>	(T2, 1, T4, 1)	<b>p318</b>	(T5, 0, 0, 1)
<b>p51</b>	(T5, T5, 1, 1)	<b>p185</b>	(T3, T2, 1, 0)	<b>p319</b>	(T, T, T3, 1)
<b>p52</b>	(T5, T5, T4, 1)	<b>p186</b>	(0, T3, T2, 1)	<b>p320</b>	(1, T, 0, 1)
<b>p53</b>	(T, T, 1, 0)	<b>p187</b>	(T4, T2, T2, 1)	<b>p321</b>	(T, T3, 0, 1)
<b>p54</b>	(0, T, T, 1)	<b>p188</b>	(T4, T, T5, 1)	<b>p322</b>	(T, T2, T5, 1)
<b>p55</b>	(T3, T, 0, 1)	<b>p189</b>	(T2, T5, 0, 1)	<b>p323</b>	(T2, T3, T3, 1)
<b>p56</b>	(T, 0, 0, 1)	<b>p190</b>	(T, 1, 1, 1)	<b>p324</b>	(1, T5, T4, 1)



<b>p57</b>	(T, T2, T3, 1)	<b>p191</b>	(T5, 1, T2, 1)	<b>p325</b>	(T, T3, 1, 0)
<b>p58</b>	(1, T, T, 1)	<b>p192</b>	(T4, T4, T, 1)	<b>p326</b>	(0, T, T3, 1)
<b>p59</b>	(T3, T5, 0, 1)	<b>p193</b>	(T3, 1, T3, 1)	<b>p327</b>	(1, T4, 0, 1)
<b>p60</b>	(T, 0, 1, 1)	<b>p194</b>	(1, 0, T3, 1)	<b>p328</b>	(T, T3, T, 1)
<b>p61</b>	(T5, 1, T, 1)	<b>p195</b>	(1, T2, T2, 1)	<b>p329</b>	(T3, T4, T, 1)
<b>p62</b>	(T3, T3, 1, 1)	<b>p196</b>	(T4, 1, T5, 1)	<b>p330</b>	(T3, 0, T3, 1)
<b>p63</b>	(T5, 0, T3, 1)	<b>p197</b>	(T2, T5, T5, 1)	<b>p331</b>	(1, 0, T2, 1)
<b>p64</b>	(1, 1, T2, 1)	<b>p198</b>	(T2, T, T, 1)	<b>p332</b>	(T4, 1, 1, 1)
<b>p65</b>	(T4, 1, T, 1)	<b>p199</b>	(T3, T2, 0, 1)	<b>p333</b>	(T5, T2, T2, 1)
<b>p66</b>	(T3, 1, 1, 1)	<b>p200</b>	(T, 0, T2, 1)	<b>p334</b>	(T4, T4, T5, 1)
<b>p67</b>	(T5, 0, T2, 1)	<b>p201</b>	(T4, T5, 1, 1)	<b>p335</b>	(T2, T5, T2, 1)
<b>p68</b>	(T4, T4, 1, 1)	<b>p202</b>	(T5, T2, T4, 1)	<b>p336</b>	(T4, T3, T3, 1)
<b>p69</b>	(T5, T2, T5, 1)	<b>p203</b>	(T5, T5, 1, 0)	<b>p337</b>	(1, T3, T4, 1)
<b>p70</b>	(T2, T2, T3, 1)	<b>p204</b>	(0, T5, T5, 1)	<b>p338</b>	(T2, T4, 1, 0)
<b>p71</b>	(1, T5, T, 1)	<b>p205</b>	(T2, 1, T, 1)	<b>p339</b>	(0, T2, T4, 1)
<b>p72</b>	(T3, T5, T2, 1)	<b>p206</b>	(T3, T2, 1, 1)	<b>p340</b>	(T5, T3, 1, 0)
<b>p73</b>	(T4, 0, T3, 1)	<b>p207</b>	(T5, 0, 1, 1)	<b>p341</b>	(0, T5, T3, 1)
<b>p74</b>	(1, T3, T2, 1)	<b>p208</b>	(T5, T5, T, 1)	<b>p342</b>	(1, T4, T5, 1)
<b>p75</b>	(T4, 1, T2, 1)	<b>p209</b>	(T3, T3, T2, 1)	<b>p343</b>	(T2, T4, T2, 1)
<b>p76</b>	(T4, T, T, 1)	<b>p210</b>	(T4, 0, T2, 1)	<b>p344</b>	(T4, T3, T4, 1)
<b>p77</b>	(T3, 1, 0, 1)	<b>p211</b>	(T4, T, 1, 1)	<b>p345</b>	(T2, T5, 1, 0)
<b>p78</b>	(T, 0, T4, 1)	<b>p212</b>	(T5, T2, 0, 1)	<b>p346</b>	(0, T2, T5, 1)
<b>p79</b>	(T4, T5, 1, 0)	<b>p213</b>	(T, T, T2, 1)	<b>p347</b>	(T2, 1, T3, 1)
<b>p80</b>	(0, T4, T5, 1)	<b>p214</b>	(T4, T5, 0, 1)	<b>p348</b>	(1, T5, T3, 1)
<b>p81</b>	(T2, 1, T2, 1)	<b>p215</b>	(T, T4, 1, 1)	<b>p349</b>	(1, T2, T5, 1)
<b>p82</b>	(T4, T3, T, 1)	<b>p216</b>	(T5, 1, T5, 1)	<b>p350</b>	(T2, T4, T3, 1)
<b>p83</b>	(T3, 1, T, 1)	<b>p217</b>	(T2, T2, T5, 1)	<b>p351</b>	(1, T5, 1, 1)
<b>p84</b>	(T3, 0, 1, 1)	<b>p218</b>	(T2, T, T3, 1)	<b>p352</b>	(T5, T, T4, 1)
<b>p85</b>	(T5, 0, T, 1)	<b>p219</b>	(1, T5, 0, 1)	<b>p353</b>	(1, 1, 0, 0)
<b>p86</b>	(T3, T3, T5, 1)	<b>p220</b>	(T, T3, 1, 1)	<b>p354</b>	(0, 1, 1, 0)
<b>p87</b>	(T2, 0, 1, 1)	<b>p221</b>	(T5, 1, T3, 1)	<b>p355</b>	(0, 0, 1, 1)
<b>p88</b>	(T5, T4, T, 1)	<b>p222</b>	(1, 1, T3, 1)	<b>p356</b>	(T5, T3, T, 1)
<b>p89</b>	(T3, T3, T3, 1)	<b>p223</b>	(1, T2, T3, 1)	<b>p357</b>	(T3, T3, T, 1)
<b>p90</b>	(1, 0, T4, 1)	<b>p224</b>	(1, T2, T, 1)	<b>p358</b>	(T3, 0, T, 1)
<b>p91</b>	(T4, 1, 1, 0)	<b>p225</b>	(T3, T5, T4, 1)	<b>p359</b>	(T3, 0, T5, 1)
<b>p92</b>	(0, T4, 1, 1)	<b>p226</b>	(T, 0, 1, 0)	<b>p360</b>	(T2, 0, T4, 1)
<b>p93</b>	(T5, T3, T5, 1)	<b>p227</b>	(0, T, 0, 1)	<b>p361</b>	(T4, T3, 1, 0)
<b>p94</b>	(T2, T2, 1, 1)	<b>p228</b>	(T, T5, 0, 1)	<b>p362</b>	(0, T4, T3, 1)
<b>p95</b>	(T5, T4, 1, 1)	<b>p229</b>	(T, T2, 1, 1)	<b>p363</b>	(1, T4, 1, 1)
<b>p96</b>	(T5, T5, T5, 1)	<b>p230</b>	(T5, 1, 1, 1)	<b>p364</b>	(T5, T, T5, 1)
<b>p97</b>	(T2, T2, T, 1)	<b>p231</b>	(T5, T5, T2, 1)	<b>p365</b>	(T2, T2, 0, 1)
<b>p98</b>	(T3, T2, T4, 1)	<b>p232</b>	(T4, T4, T3, 1)	<b>p366</b>	(T, 1, T2, 1)
<b>p99</b>	(T5, 0, 1, 0)	<b>p233</b>	(1, T3, 1, 1)	<b>p367</b>	(T4, T5, T, 1)
<b>p100</b>	(0, T5, 0, 1)	<b>p234</b>	(T5, T, T3, 1)	<b>p368</b>	(T3, 1, T2, 1)
<b>p101</b>	(T, T5, 1, 1)	<b>p235</b>	(1, 1, 0, 1)	<b>p369</b>	(T4, 0, T, 1)
<b>p102</b>	(T5, 1, T4, 1)	<b>p236</b>	(T, T3, T4, 1)	<b>p370</b>	(T3, 1, T5, 1)
<b>p103</b>	(T3, T3, 1, 0)	<b>p237</b>	(T2, T3, 1, 0)	<b>p371</b>	(T2, 0, T5, 1)
<b>p104</b>	(0, T3, T3, 1)	<b>p238</b>	(0, T2, T3, 1)	<b>p372</b>	(T2, T, T4, 1)
<b>p105</b>	(1, T4, T4, 1)	<b>p239</b>	(1, T4, T, 1)	<b>p373</b>	(T, 1, 0, 0)
<b>p106</b>	(1, T2, 1, 0)	<b>p240</b>	(T3, T5, T3, 1)	<b>p374</b>	(0, T, 1, 0)
<b>p107</b>	(0, 1, T2, 1)	<b>p241</b>	(1, 0, T5, 1)	<b>p375</b>	(0, 0, T, 1)
<b>p108</b>	(T4, T2, T, 1)	<b>p242</b>	(T2, T4, T4, 1)	<b>p376</b>	(T3, T, T5, 1)
<b>p109</b>	(T3, 1, T4, 1)	<b>p243</b>	(1, T5, 1, 0)	<b>p377</b>	(T2, 0, 0, 1)
<b>p110</b>	(T3, 0, 1, 0)	<b>p244</b>	(0, 1, T5, 1)	<b>p378</b>	(T, 1, T3, 1)
<b>p111</b>	(0, T3, 0, 1)	<b>p245</b>	(T2, 1, T5, 1)	<b>p379</b>	(1, T, T3, 1)
<b>p112</b>	(T, T5, T5, 1)	<b>p246</b>	(T2, T, T5, 1)	<b>p380</b>	(1, T2, 0, 1)



<b>p113</b>	(T2, T3, T, 1)	<b>p247</b>	(T2, T, 0, 1)	<b>p381</b>	(T, T3, T2, 1)
<b>p114</b>	(T3, T2, T, 1)	<b>p248</b>	(T, 1, 0, 1)	<b>p382</b>	(T4, T5, T2, 1)
<b>p115</b>	(T3, 0, T4, 1)	<b>p249</b>	(T, T2, T4, 1)	<b>p383</b>	(T4, T, T3, 1)
<b>p116</b>	(T4, 0, 1, 0)	<b>p250</b>	(T5, 1, 1, 0)	<b>p384</b>	(1, T3, 0, 1)
<b>p117</b>	(0, T4, 0, 1)	<b>p251</b>	(0, T5, 1, 1)	<b>p385</b>	(T, T3, T5, 1)
<b>p118</b>	(T, T5, T, 1)	<b>p252</b>	(T5, T3, T4, 1)	<b>p386</b>	(T2, T3, 1, 1)
<b>p119</b>	(T3, T4, T2, 1)	<b>p253</b>	(T2, T2, 1, 0)	<b>p387</b>	(T5, T4, T3, 1)
<b>p120</b>	(T4, 0, T4, 1)	<b>p254</b>	(0, T2, T2, 1)	<b>p388</b>	(1, 1, 1, 1)
<b>p121</b>	(T4, T, 1, 0)	<b>p255</b>	(T4, T2, T5, 1)	<b>p389</b>	(T5, T, T2, 1)
<b>p122</b>	(0, T4, T, 1)	<b>p256</b>	(T2, T5, T3, 1)	<b>p390</b>	(T4, T4, 0, 1)
<b>p123</b>	(T3, T, T3, 1)	<b>p257</b>	(1, T5, T5, 1)	<b>p391</b>	(T, T4, T, 1)
<b>p124</b>	(1, 0, 0, 1)	<b>p258</b>	(T2, T4, T, 1)	<b>p392</b>	(T3, T4, T3, 1)
<b>p125</b>	(T, T3, T3, 1)	<b>p259</b>	(T3, T2, T3, 1)	<b>p393</b>	(1, 0, 1, 1)
<b>p126</b>	(1, T, T4, 1)	<b>p260</b>	(1, 0, T, 1)	<b>p394</b>	(T5, T, T, 1)
<b>p127</b>	(T4, 1, 0, 0)	<b>p261</b>	(T3, T5, T5, 1)	<b>p395</b>	(T3, T3, 0, 1)
<b>p128</b>	(0, T4, 1, 0)	<b>p262</b>	(T2, 0, T, 1)	<b>p396</b>	(T, 0, T5, 1)
<b>p129</b>	(0, 0, T4, 1)	<b>p263</b>	(T3, T2, T5, 1)	<b>p397</b>	(T2, T3, T4, 1)
<b>p130</b>	(T4, T2, 1, 0)	<b>p264</b>	(T2, 0, T3, 1)	<b>p398</b>	(T2, T, 1, 0)
<b>p131</b>	(0, T4, T2, 1)	<b>p265</b>	(1, T5, T2, 1)	<b>p399</b>	(0, T2, T, 1)
<b>p132</b>	(T4, T2, T4, 1)	<b>p266</b>	(T4, 1, T3, 1)	<b>p400</b>	(T3, T, T4, 1)
<b>p133</b>	(T5, T2, 1, 0)	<b>p267</b>	(1, T3, T3, 1)	<b>p269</b>	(T5, T, 1, 0)
<b>p134</b>	(0, T5, T2, 1)	<b>p268</b>	(1, T2, T4, 1)	<b>p270</b>	(0, T5, T, 1)

With selecting the point in  $PG(3,7)$  such that the third cord innate equal to zero this means it belongs to  $l_0 = V(Z)$  such that all in  $F_7 \setminus \{0\}$ ; therefore,  $p_i = i = 1, 2, \dots, 400$  we obtain  $l_1 = \{1, 2, 46, 127, 158, 181, 353, 373\}$  where

$$l_i = l_1 \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ T^2 & 1 & T^4 & T \end{pmatrix}^i, i = 1, 2, \dots, 400,$$

**Table(2) The lines of  $PG(3,7)$  are:**

<b>L1</b>	1	2	46	127	158	181	353	373
<b>L2</b>	2	3	47	128	159	182	354	374
<b>L3</b>	3	4	48	129	160	183	355	375
<b>L4</b>	4	5	49	130	161	184	356	376
<b>L5</b>	5	6	50	131	162	185	357	377
<b>L6</b>	6	7	51	132	163	186	358	378
<b>L7</b>	7	8	52	133	164	187	359	379
<b>L8</b>	8	9	53	134	165	188	360	380
<b>L9</b>	9	10	54	135	166	189	361	381
<b>L10</b>	10	11	55	136	167	190	362	382
<b>L11</b>	11	12	56	137	168	191	363	383
<b>L12</b>	12	13	57	138	169	192	364	384
<b>L13</b>	13	14	58	139	170	193	365	385
<b>L14</b>	14	15	59	140	171	194	366	386
<b>L15</b>	15	16	60	141	172	195	367	387
<b>L16</b>	16	17	61	142	173	196	368	388
<b>L17</b>	17	18	62	143	174	197	369	389
<b>L18</b>	18	19	63	144	175	198	370	390
<b>L19</b>	19	20	64	145	176	199	371	391



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<b>L20</b>	20	21	65	146	177	200	372	392
<b>L21</b>	21	22	66	147	178	201	373	393
<b>L22</b>	22	23	67	148	179	202	374	394
<b>L23</b>	23	24	68	149	180	203	375	395
<b>L24</b>	24	25	69	150	181	204	376	396
<b>L25</b>	25	26	70	151	182	205	377	397
<b>L26</b>	26	27	71	152	183	206	378	398
<b>L27</b>	27	28	72	153	184	207	379	399
<b>L28</b>	28	29	73	154	185	208	380	400
<b>L29</b>	29	30	74	155	186	209	381	1
<b>L30</b>	30	31	75	156	187	210	382	2
<b>L31</b>	31	32	76	157	188	211	383	3
<b>L32</b>	32	33	77	158	189	212	384	4
<b>L33</b>	33	34	78	159	190	213	385	5
<b>L34</b>	34	35	79	160	191	214	386	6
<b>L35</b>	35	36	80	161	192	215	387	7
<b>L36</b>	36	37	81	162	193	216	388	8
<b>L37</b>	37	38	82	163	194	217	389	9
<b>L38</b>	38	39	83	164	195	218	390	10
<b>L39</b>	39	40	84	165	196	219	391	11
<b>L40</b>	40	41	85	166	197	220	392	12
<b>L41</b>	41	42	86	167	198	221	393	13
<b>L42</b>	42	43	87	168	199	222	394	14
<b>L43</b>	43	44	88	169	200	223	395	15
<b>L44</b>	44	45	89	170	201	224	396	16
<b>L45</b>	45	46	90	171	202	225	397	17
<b>L46</b>	46	47	91	172	203	226	398	18
<b>L47</b>	47	48	92	173	204	227	399	19
<b>L48</b>	48	49	93	174	205	228	400	20
<b>L49</b>	49	50	94	175	206	229	1	21
<b>L50</b>	50	51	95	176	207	230	2	22
<b>L51</b>	51	52	96	177	208	231	3	23
<b>L52</b>	52	53	97	178	209	232	4	24
<b>L53</b>	53	54	98	179	210	233	5	25
<b>L54</b>	54	55	99	180	211	234	6	26
<b>L55</b>	55	56	100	181	212	235	7	27
<b>L56</b>	56	57	101	182	213	236	8	28
<b>L57</b>	57	58	102	183	214	237	9	29
<b>L58</b>	58	59	103	184	215	238	10	30
<b>L59</b>	59	60	104	185	216	239	11	31
<b>L60</b>	60	61	105	186	217	240	12	32
<b>L61</b>	61	62	106	187	218	241	13	33
<b>L62</b>	62	63	107	188	219	242	14	34
<b>L63</b>	63	64	108	189	220	243	15	35
<b>L64</b>	64	65	109	190	221	244	16	36
<b>L65</b>	65	66	110	191	222	245	17	37
<b>L66</b>	66	67	111	192	223	246	18	38
<b>L67</b>	67	68	112	193	224	247	19	39
<b>L68</b>	68	69	113	194	225	248	20	40
<b>L69</b>	69	70	114	195	226	249	21	41
<b>L70</b>	70	71	115	196	227	250	22	42
<b>L71</b>	71	72	116	197	228	251	23	43
<b>L72</b>	72	73	117	198	229	252	24	44
<b>L73</b>	73	74	118	199	230	253	25	45
<b>L74</b>	74	75	119	200	231	254	26	46
<b>L75</b>	75	76	120	201	232	255	27	47

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<b>L76</b>	76	77	121	202	233	256	28	48
<b>L77</b>	77	78	122	203	234	257	29	49
<b>L78</b>	78	79	123	204	235	258	30	50
<b>L79</b>	79	80	124	205	236	259	31	51
<b>L80</b>	80	81	125	206	237	260	32	52
<b>L81</b>	81	82	126	207	238	261	33	53
<b>L82</b>	82	83	127	208	239	262	34	54
<b>L83</b>	83	84	128	209	240	263	35	55
<b>L84</b>	84	85	129	210	241	264	36	56
<b>L85</b>	85	86	130	211	242	265	37	57
<b>L86</b>	86	87	131	212	243	266	38	58
<b>L87</b>	87	88	132	213	244	267	39	59
<b>L88</b>	88	89	133	214	245	268	40	60
<b>L89</b>	89	90	134	215	246	269	41	61
<b>L90</b>	90	91	135	216	247	270	42	62
<b>L91</b>	91	92	136	217	248	271	43	63
<b>L92</b>	92	93	137	218	249	272	44	64
<b>L93</b>	93	94	138	219	250	273	45	65
<b>L94</b>	94	95	139	220	251	274	46	66
<b>L95</b>	95	96	140	221	252	275	47	67
<b>L96</b>	96	97	141	222	253	276	48	68
<b>L97</b>	97	98	142	223	254	277	49	69
<b>L98</b>	98	99	143	224	255	278	50	70
<b>L99</b>	99	100	144	225	256	279	51	71
<b>L100</b>	100	101	145	226	257	280	52	72
<b>L101</b>	101	102	146	227	258	281	53	73
<b>L102</b>	102	103	147	228	259	282	54	74
<b>L103</b>	103	104	148	229	260	283	55	75
<b>L104</b>	104	105	149	230	261	284	56	76
<b>L105</b>	105	106	150	231	262	285	57	77
<b>L106</b>	106	107	151	232	263	286	58	78
<b>L107</b>	107	108	152	233	264	287	59	79
<b>L108</b>	108	109	153	234	265	288	60	80
<b>L109</b>	109	110	154	235	266	289	61	81
<b>L110</b>	110	111	155	236	267	290	62	82
<b>L111</b>	111	112	156	237	268	291	63	83
<b>L112</b>	112	113	157	238	269	292	64	84
<b>L113</b>	113	114	158	239	270	293	65	85
<b>L114</b>	114	115	159	240	271	294	66	86
<b>L115</b>	115	116	160	241	272	295	67	87
<b>L116</b>	116	117	161	242	273	296	68	88
<b>L117</b>	117	118	162	243	274	297	69	89
<b>L318</b>	318	319	363	44	75	98	270	290
<b>L319</b>	319	320	364	45	76	99	271	291
<b>L320</b>	320	321	365	46	77	100	272	292
<b>L321</b>	321	322	366	47	78	101	273	293
<b>L322</b>	322	323	367	48	79	102	274	294
<b>L323</b>	323	324	368	49	80	103	275	295
<b>L324</b>	324	325	369	50	81	104	276	296
<b>L325</b>	325	326	370	51	82	105	277	297
<b>L326</b>	326	327	371	52	83	106	278	298
<b>L327</b>	327	328	372	53	84	107	279	299
<b>L328</b>	328	329	373	54	85	108	280	300
<b>L329</b>	329	330	374	55	86	109	281	301



L330	330	331	375	56	87	110	282	302
L331	331	332	376	57	88	111	283	303
L332	332	333	377	58	89	112	284	304
L333	333	334	378	59	90	113	285	305
L334	334	335	379	60	91	114	286	306
L335	335	336	380	61	92	115	287	307
L336	336	337	381	62	93	116	288	308
L337	337	338	382	63	94	117	289	309
L338	338	339	383	64	95	118	290	310
L339	339	340	384	65	96	119	291	311
L340	340	341	385	66	97	120	292	312
L341	341	342	386	67	98	121	293	313
L342	342	343	387	68	99	122	294	314
L343	343	344	388	69	100	123	295	315
L344	344	345	389	70	101	124	296	316
L345	345	346	390	71	102	125	297	317
L346	346	347	391	72	103	126	298	318
L347	347	348	392	73	104	127	299	319
L348	348	349	393	74	105	128	300	320
L349	349	350	394	75	106	129	301	321
L350	350	351	395	76	107	130	302	322
L351	351	352	396	77	108	131	303	323
L352	352	353	397	78	109	132	304	324
L353	353	354	398	79	110	133	305	325
L354	354	355	399	80	111	134	306	326
L355	355	356	400	81	112	135	307	327
L356	356	357	1	82	113	136	308	328
L357	357	358	2	83	114	137	309	329
L358	358	359	3	84	115	138	310	330
L359	359	360	4	85	116	139	311	331
L360	360	361	5	86	117	140	312	332
L361	361	362	6	87	118	141	313	333
L362	362	363	7	88	119	142	314	334
L363	363	364	8	89	120	143	315	335
L364	364	365	9	90	121	144	316	336
L365	365	366	10	91	122	145	317	337
L366	366	367	11	92	123	146	318	338
L367	367	368	12	93	124	147	319	339
L368	368	369	13	94	125	148	320	340
L369	369	370	14	95	126	149	321	341
L370	370	371	15	96	127	150	322	342
L371	371	372	16	97	128	151	323	343
L372	372	373	17	98	129	152	324	344
L373	373	374	18	99	130	153	325	345
L374	374	375	19	100	131	154	326	346
L375	375	376	20	101	132	155	327	347
L376	376	377	21	102	133	156	328	348
L377	377	378	22	103	134	157	329	349





L378	378	379	23	104	135	158	330	350
L379	379	380	24	105	136	159	331	351
L380	380	381	25	106	137	160	332	352
L381	381	382	26	107	138	161	333	353
L382	382	383	27	108	139	162	334	354
L383	383	384	28	109	140	163	335	355
L384	384	385	29	110	141	164	336	356
L385	385	386	30	111	142	165	337	357
L386	386	387	31	112	143	166	338	358
L387	387	388	32	113	144	167	339	359
L388	388	389	33	114	145	168	340	360
L389	389	390	34	115	146	169	341	361
L390	390	391	35	116	147	170	342	362
L391	391	392	36	117	148	171	343	363
L392	392	393	37	118	149	172	344	364
L393	393	394	38	119	150	173	345	365
L394	394	395	39	120	151	174	346	366
L395	395	396	40	121	152	175	347	367
L396	396	397	41	122	153	176	348	368
L397	397	398	42	123	154	177	349	369
L398	398	399	43	124	155	178	350	370
L399	399	400	44	125	156	179	351	371
L400	400	1	45	126	157	180	352	372

With selecting the point in  $PG(3,7)$  such that the fourth cord innate equal to zero this means it belongs to  $plane_1 = V(Z)$  such that all in  $F_7 \setminus \{0\}$ ; therefore,  $p_i = i = 1, 2, \dots, 400$  we obtain  $plane_1 =$

{1,2,3,12,18,20,36,39,44,46,47,53,79,91,99,103,106,110,116,121,127,128,130,142,153,158,159,163,167,172,181,182,

185,203,226,237,243,250,253,269,284,303,305,307,315,325,338,340,345,353,354,361,373,374,398}

where

$$plane_i = plane_1 \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ T^2 & 1 & T^4 & T \end{pmatrix}^i, i = 1, 2, \dots, 400$$

**Table (3) The plane of  $PG(3,7)$  are:**

plane 1  
1,2,3,12,18,20,36,39,44,46,47,53,79,91,99,103,106,110,116,121,127,128,130,142,153,158,159,163,167,172,181,182,185,203,226,237,243,250,253,269,284,303,305,307,315,325,338,340,345,353,354,361,373,374,398

plane 2  
2,3,4,13,19,21,37,40,45,47,48,54,80,92,100,104,107,111,117,122,128,129,131,143,154,159,160,164,168,173,182,183,186,204,227,238,244,251,254,270,285,304,306,308,316,326,339,341,346,354,355,362,374,375,399



plane 3

3,4,5,14,20,22,38,41,46,48,49,55,81,93,101,105,108,112,118,123,129,130,132,144,155,160,161,165,169,174,183,184,187,205,228,239,245,252,255,271,286,305,307,309,317,327,340,342,347,355,356,363,375,376,400

.....

plane 138

138,139,140,149,155,157,173,176,181,183,184,190,216,228,236,240,243,247,253,258,264,265,267,279,290,295,296,300,304,309,318,319,322,340,363,374,380,387,390,6,21,40,42,44,52,62,75,77,82,90,91,98,110,111,135

plane 139

139,140,141,150,156,158,174,177,182,184,185,191,217,229,237,241,244,248,254,259,265,266,268,280,291,296,297,301,305,310,319,320,323,341,364,375,381,388,391,7,22,41,43,45,53,63,76,78,83,91,92,99,111,112,136

plane 140

140,141,142,151,157,159,175,178,183,185,186,192,218,230,238,242,245,249,255,260,266,267,269,281,292,297,298,302,306,311,320,321,324,342,365,376,382,389,392,8,23,42,44,46,54,64,77,79,84,92,93,100,112,113,137

.....

plane 225

225,226,227,236,242,244,260,263,268,270,271,277,303,315,323,327,330,334,340,345,351,352,354,366,377,382,383,387,391,396,5,6,9,27,50,61,67,74,77,93,108,127,129,131,139,149,162,164,169,177,178,185,197,198,222

plane 226

226,227,228,237,243,245,261,264,269,271,272,278,304,316,324,328,331,335,341,346,352,353,355,367,378,383,384,388,392,397,6,7,10,28,51,62,68,75,78,94,109,128,130,132,140,150,163,165,170,178,179,186,198,199,223

plane 227

227,228,229,238,244,246,262,265,270,272,273,279,305,317,325,329,332,336,342,347,353,354,356,368,379,384,385,389,393,398,7,8,11,29,52,63,69,76,79,95,110,129,131,133,141,151,164,166,171,179,180,187,199,200,224

.....

plane 398

398,399,400,9,15,17,33,36,41,43,44,50,76,88,96,100,103,107,113,118,124,125,127,139,150,155,156,160,164,169,178,179,182,200,223,234,240,247,250,266,281,300,302,304,312,322,335,337,342,350,351,358,370,371,395

plane 399

399,400,1,10,16,18,34,37,42,44,45,51,77,89,97,101,104,108,114,119,125,126,128,140,151,156,157,161,165,170,179,180,183,201,224,235,241,248,251,267,282,301,303,305,313,323,336,338,343,351,352,359,371,372,396

plane 400

400,1,2,11,17,19,35,38,43,45,46,52,78,90,98,102,105,109,115,120,126,127,129,141,152,157,158,162,166,171,180,181,184,202,225,236,242,249,252,268,283,302,304,306,314,324,337,339,344,352,353,360,372,373,397

**3-New Results :**

in the following theorem the parameters n,m,d are constructed.

**Theorem 3,1 :** The projective space of order 7 is a code with a parameter [ n=400,M=7<sup>396</sup>,d=57 ].

Proof: the plane  $\pi_7$  has an incidence matrix  $A = (a_{ij})$  where  $a_{ij} = \begin{cases} 1 & \text{if } p_i \in \text{plane}_j \\ 0 & \text{if } p_i \notin \text{plane}_j \end{cases}$  Then we

have the incider Table(3,1).

**Table (3,1)**





$$m_i = [0,0,0,0,0,0, \dots \dots \dots 0,0,0,0,0]$$

$$v_i = [1,1,1,1,1,1, \dots \dots \dots 1,1,1,1,1]$$

$$h_i = [T, T, T, T, T, T, \dots \dots \dots T, T, T, T, T]$$

$$k_i = [T^2, T^2, T^2, T^2, T^2, \dots \dots \dots T^2, T^2, T^2, T^2]$$

$$o_i = [T^3, T^3, T^3, T^3, T^3, \dots \dots \dots T^3, T^3, T^3, T^3]$$

$$w_i = [T^4, T^4, T^4, T^4, T^4, \dots \dots \dots T^4, T^4, T^4, T^4]$$

$$s_i = [T^5, T^5, T^5, T^5, T^5, \dots \dots \dots T^5, T^5, T^5, T^5]$$

Combine  $v_i$  with a binary system projective matrix we get a zero-one

$$matrix\ v_i = [1,1,1,1,1,1, \dots \dots \dots 1,1,1,1,1]$$

Let  $a_i = v_i + PLAN_i \quad 1 \leq i \leq 400$

Table (3, 2) Now the table of  $a_i$

+	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	...	P399	P400
a1	T4	T4	T4	1	1	1	1	1	1	1	...	1	1
a2	1	T4	T4	T4	1	1	1	1	1	1	...	T4	1
a3	1	1	T4	T4	T4	1	1	1	1	1	...	1	T4
a4	T4	1	1	T4	T4	T4	1	1	1	1	...	1	1
a5	1	T4	1	1	T4	T4	T4	1	1	1	...	1	1
a6	1	1	T4	1	1	T4	T4	T4	1	1	...	1	1
a7	1	1	1	T4	1	1	T4	T4	T4	1	...	T4	1
a8	1	1	1	1	T4	1	1	T4	T4	T4	...	1	1
a9	1	1	1	1	1	T4	1	1	T4	T4	...	1	1
a10	1	1	1	1	1	1	T4	1	1	T4	...	1	1
a11	1	1	1	1	1	1	1	T4	1	1	...	1	1
a12	1	1	1	1	1	1	1	1	T4	1	...	1	1
a13	1	1	1	1	1	1	1	1	1	T4	...	1	1
a14	1	1	1	1	1	1	1	1	1	1	...	1	1
a15	1	1	1	1	1	1	1	1	1	1	...	1	1
a16	1	1	1	1	1	1	1	1	1	1	...	1	1
a17	1	1	1	1	1	1	1	1	1	1	...	1	1
a18	1	1	1	1	1	1	1	1	1	1	...	1	1
a19	1	1	1	1	1	1	1	1	1	1	...	1	1
...	...	...	...	...	...	...	...	...	...	...	...	...	...
a381	1	1	1	1	1	1	1	1	1	1	...	1	T4
a382	T4	1	1	1	1	1	1	1	1	1	...	T4	1
a383	1	T4	1	1	1	1	1	1	1	1	...	1	T4
a384	T4	1	T4	1	1	1	1	1	1	1	...	1	1
a385	1	T4	1	T4	1	1	1	1	1	1	...	1	1
a386	1	1	T4	1	T4	1	1	1	1	1	...	1	1
a387	1	1	1	T4	1	T4	1	1	1	1	...	1	1
a388	1	1	1	1	T4	1	T4	1	1	1	...	T4	1
a389	1	1	1	1	1	T4	1	T4	1	1	...	1	T4
a390	T4	1	1	1	1	1	T4	1	T4	1	...	1	1
a391	1	T4	1	1	1	1	1	T4	1	T4	...	1	1
a392	1	1	T4	1	1	1	1	1	T4	1	...	1	1
a393	1	1	1	T4	1	1	1	1	1	T4	...	1	1
a394	1	1	1	1	T4	1	1	1	1	1	...	1	1
a395	1	1	1	1	1	T4	1	1	1	1	...	1	1
a396	1	1	1	1	1	1	T4	1	1	1	...	1	1
a397	1	1	1	1	1	1	1	T4	1	1	...	T4	1
a398	1	1	1	1	1	1	1	1	T4	1	...	T4	T4
a399	T4	1	1	1	1	1	1	1	1	T4	...	T4	T4
a400	T4	T4	1	1	1	1	1	1	1	1	...	1	T4

Combine  $h_i$  with a binary system projective matrix we get a zero-one



matrix  $h_i = [ T, T, T, T, T, T, T, \dots \dots \dots T, T, T, T, T ]$

Let  $b_i = h_i + PLAN_i \quad 1 \leq i \leq 400$

Table(3,3) Now the table of  $b_i$

+	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	...	P399	P400
b1	T3	T3	T3	T	T	T	T	T	T	T	...	T	T
b2	T	T3	T3	T3	T	T	T	T	T	T	...	T3	T
b3	T	T	T3	T3	T3	T	T	T	T	T	...	T	T3
b4	T3	T	T	T3	T3	T3	T	T	T	T	...	T	T
b5	T	T3	T	T	T3	T3	T3	T	T	T	...	T	T
b6	T	T	T3	T	T	T3	T3	T3	T	T	...	T	T
b7	T	T	T	T3	T	T	T3	T3	T3	T	...	T3	T
b8	T	T	T	T	T3	T	T	T3	T3	T3	...	T	T
b9	T	T	T	T	T	T3	T	T	T3	T3	...	T	T
b10	T	T	T	T	T	T	T3	T	T	T3	...	T	T
b11	T	T	T	T	T	T	T	T3	T	T	...	T	T
b12	T	T	T	T	T	T	T	T	T3	T	...	T	T
b13	T	T	T	T	T	T	T	T	T	T3	...	T	T
b14	T	T	T	T	T	T	T	T	T	T	...	T	T
b15	T	T	T	T	T	T	T	T	T	T	...	T	T
b16	T	T	T	T	T	T	T	T	T	T	...	T	T
b17	T	T	T	T	T	T	T	T	T	T	...	T	T
b18	T	T	T	T	T	T	T	T	T	T	...	T	T
b19	T	T	T	T	T	T	T	T	T	T	...	T	T
...	...	...	...	...	...	...	...	...	...	...	...	...	...
b381	T	T	T	T	T	T	T	T	T	T	...	T	T3
b382	T3	T	T	T	T	T	T	T	T	T	...	T3	T
b383	T	T3	T	T	T	T	T	T	T	T	...	T	T3
b384	T3	T	T3	T	T	T	T	T	T	T	...	T	T
b385	T	T3	T	T3	T	T	T	T	T	T	...	T	T
b386	T	T	T3	T	T3	T	T	T	T	T	...	T	T
b387	T	T	T	T3	T	T3	T	T	T	T	...	T	T
b388	T	T	T	T	T3	T	T3	T	T	T	...	T3	T
b389	T	T	T	T	T	T3	T	T3	T	T	...	T	T3
b390	T3	T	T	T	T	T	T3	T	T3	T	...	T	T
b391	T	T3	T	T	T	T	T	T3	T	T3	...	T	T
b392	T	T	T3	T	T	T	T	T	T3	T	...	T	T
b393	T	T	T	T3	T	T	T	T	T	T3	...	T	T
b394	T	T	T	T	T3	T	T	T	T	T	...	T	T
b395	T	T	T	T	T	T3	T	T	T	T	...	T	T
b396	T	T	T	T	T	T	T3	T	T	T	...	T	T
b397	T	T	T	T	T	T	T	T3	T	T	...	T3	T
b398	T	T	T	T	T	T	T	T	T3	T	...	T3	T3
b399	T3	T	T	T	T	T	T	T	T	T3	...	T3	T3
b400	T3	T3	T	T	T	T	T	T	T	T	...	T	T3

Combine  $k_i$  with a binary system projective matrix we get a zero-one

matrix  $k_i = [ T^2, T^2, T^2, T^2, T^2, \dots \dots \dots T^2, T^2, T^2, T^2 ]$

Let  $c_i = k_i + PLAN_i \quad 1 \leq i \leq 400$

Table(3,4) Now the table of  $c_i$

+	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	...	P399	P400
c1	T	T	T	T2	T2	T2	T2	T2	T2	T2	...	T2	T2
c2	T2	T	T	T	T2	T2	T2	T2	T2	T2	...	T	T2
c3	T2	T2	T	T	T	T2	T2	T2	T2	T2	...	T2	T
c4	T	T2	T2	T	T	T	T2	T2	T2	T2	...	T2	T2
c5	T2	T	T2	T2	T	T	T	T2	T2	T2	...	T2	T2
c6	T2	T2	T	T2	T2	T	T	T	T2	T2	...	T2	T2
c7	T2	T2	T2	T	T2	T2	T	T	T	T2	...	T	T2
c8	T2	T2	T2	T2	T	T2	T2	T	T	T	...	T2	T2
c9	T2	T2	T2	T2	T2	T	T2	T2	T	T	...	T2	T2
c10	T2	T2	T2	T2	T2	T2	T	T2	T2	T	...	T2	T2
c11	T2	T2	T2	T2	T2	T2	T2	T	T2	T2	...	T2	T2
c12	T2	T2	T2	T2	T2	T2	T2	T	T2	T2	...	T2	T2



c13	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	T	...	T2	T2
c14	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	...	T2	T2
c15	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	...	T2	T2
c16	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	...	T2	T2
c17	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	...	T2	T2
c18	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	...	T2	T2
c19	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	...	T2	T2
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
c381	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	...	T2	T
c382	T	T2	T2	T2	T2	T2	T2	T2	T2	T2	T2	...	T	T2
c383	T2	T	T2	T2	T2	T2	T2	T2	T2	T2	T2	...	T2	T
c384	T	T2	T	T2	T2	T2	T2	T2	T2	T2	T2	...	T2	T2
c385	T2	T	T2	T	T2	T2	T2	T2	T2	T2	T2	...	T2	T2
c386	T2	T2	T	T2	T	T2	T2	T2	T2	T2	T2	...	T2	T2
c387	T2	T2	T2	T	T2	T	T2	T2	T2	T2	T2	...	T2	T2
c388	T2	T2	T2	T2	T	T2	T	T2	T2	T2	T2	...	T	T2
c389	T2	T2	T2	T2	T2	T	T2	T	T2	T2	T2	...	T2	T
c390	T	T2	T2	T2	T2	T2	T	T2	T	T2	T2	...	T2	T2
c391	T2	T	T2	T2	T2	T2	T2	T	T2	T	T2	...	T2	T2
c392	T2	T2	T	T2	T2	T2	T2	T2	T	T2	T2	...	T2	T2
c393	T2	T2	T2	T	T2	T2	T2	T2	T2	T	T2	...	T2	T2
c394	T2	T2	T2	T2	T	T2	T2	T2	T2	T2	T2	...	T2	T2
c395	T2	T2	T2	T2	T2	T	T2	T2	T2	T2	T2	...	T2	T2
c396	T2	T2	T2	T2	T2	T2	T	T2	T2	T2	T2	...	T2	T2
c397	T2	T2	T2	T2	T2	T2	T2	T	T2	T2	T2	...	T	T2
c398	T2	T2	T2	T2	T2	T2	T2	T2	T	T2	T2	...	T	T
c399	T	T2	T2	T2	T2	T2	T2	T2	T2	T	T2	...	T	T
c400	T	T	T2	T2	T2	T2	T2	T2	T2	T2	T2	...	T2	T

Combine  $o_i$  with a binary system projective matrix we get a zero-one

$$\text{matrix } o_i = [T^3, T^3, T^3, T^3, T^3, \dots, T^3, T^3, T^3, T^3]$$

Let  $d_i = o_i + PLAN_i \quad 1 \leq i \leq 400$

Table (3,5) Now the table of  $d_i$

+	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	...	P399	P400
d1	0	0	0	T3	T3	T3	T3	T3	T3	T3	...	T3	T3
d2	T3	0	0	0	T3	T3	T3	T3	T3	T3	...	0	T3
d3	T3	T3	0	0	0	T3	T3	T3	T3	T3	...	T3	0
d4	0	T3	T3	0	0	0	T3	T3	T3	T3	...	T3	T3
d5	T3	0	T3	T3	0	0	0	T3	T3	T3	...	T3	T3
d6	T3	T3	0	T3	T3	0	0	0	T3	T3	...	T3	T3
d7	T3	T3	T3	0	T3	T3	0	0	0	T3	...	0	T3
d8	T3	T3	T3	T3	0	T3	T3	0	0	0	...	T3	T3
d9	T3	T3	T3	T3	T3	0	T3	T3	0	0	...	T3	T3
d10	T3	T3	T3	T3	T3	T3	0	T3	T3	0	...	T3	T3
d11	T3	T3	T3	T3	T3	T3	T3	0	T3	T3	...	T3	T3
d12	T3	T3	T3	T3	T3	T3	T3	T3	0	T3	...	T3	T3
d13	T3	T3	T3	T3	T3	T3	T3	T3	T3	0	...	T3	T3
d14	T3	T3	T3	T3	T3	T3	T3	T3	T3	T3	...	T3	T3
d15	T3	T3	T3	T3	T3	T3	T3	T3	T3	T3	...	T3	T3
d16	T3	T3	T3	T3	T3	T3	T3	T3	T3	T3	...	T3	T3
d17	T3	T3	T3	T3	T3	T3	T3	T3	T3	T3	...	T3	T3
d18	T3	T3	T3	T3	T3	T3	T3	T3	T3	T3	...	T3	T3
d19	T3	T3	T3	T3	T3	T3	T3	T3	T3	T3	...	T3	T3
...	...	...	...	...	...	...	...	...	...	...	...	...	...
d381	T3	T3	T3	T3	T3	T3	T3	T3	T3	T3	...	T3	0
d382	0	T3	T3	T3	T3	T3	T3	T3	T3	T3	...	0	T3
d383	T3	0	T3	T3	T3	T3	T3	T3	T3	T3	...	T3	0
d384	0	T3	0	T3	T3	T3	T3	T3	T3	T3	...	T3	T3
d385	T3	0	T3	0	T3	T3	T3	T3	T3	T3	...	T3	T3
d386	T3	T3	0	T3	0	T3	T3	T3	T3	T3	...	T3	T3
d387	T3	T3	T3	0	T3	0	T3	T3	T3	T3	...	T3	T3
d388	T3	T3	T3	T3	0	T3	0	T3	T3	T3	...	0	T3
d389	T3	T3	T3	T3	T3	0	T3	0	T3	T3	...	T3	0
d390	0	T3	T3	T3	T3	T3	0	T3	0	T3	...	T3	T3
d391	T3	0	T3	T3	T3	T3	0	T3	0	...	T3	T3	T3
d392	T3	T3	0	T3	T3	T3	T3	0	T3	...	T3	T3	T3



d393	T3	T3	T3	0	T3	T3	T3	T3	T3	0	...	T3	T3
d394	T3	T3	T3	T3	0	T3	T3	T3	T3	T3	...	T3	T3
d395	T3	T3	T3	T3	T3	0	T3	T3	T3	T3	...	T3	T3
d396	T3	T3	T3	T3	T3	T3	0	T3	T3	T3	...	T3	T3
d397	T3	T3	T3	T3	T3	T3	T3	0	T3	T3	...	0	T3
d398	T3	T3	T3	T3	T3	T3	T3	T3	0	T3	...	0	0
d399	0	T3	T3	T3	T3	T3	T3	T3	T3	0	...	0	0
d400	0	0	T3	T3	T3	T3	T3	T3	T3	T3	...	T3	0

Combine  $w_i$  with a binary system projective matrix we get a zero-one

$$matrix\ w_i = [T^4, T^4, T^4, T^4, T^4, \dots, \dots, \dots, T^4, T^4, T^4, T^4]$$

Let  $e_i = w_i + PLAN_i \quad 1 \leq i \leq 400$

Table(3,6) Now the table of  $e_i$

'	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	...	P399	P400
e1	T5	T5	T5	T4	T4	T4	T4	T4	T4	T4	T4	...	T4	T4
e2	T4	T5	T5	T5	T4	T4	T4	T4	T4	T4	T4	...	T5	T4
e3	T4	T4	T5	T5	T5	T4	T4	T4	T4	T4	T4	...	T4	T5
e4	T5	T4	T4	T5	T5	T5	T4	T4	T4	T4	T4	...	T4	T4
e5	T4	T5	T4	T4	T5	T5	T5	T4	T4	T4	T4	...	T4	T4
e6	T4	T4	T5	T4	T4	T5	T5	T5	T4	T4	T4	...	T4	T4
e7	T4	T4	T4	T5	T4	T4	T5	T5	T5	T4	T4	...	T5	T4
e8	T4	T4	T4	T4	T5	T4	T4	T5	T5	T5	T4	...	T4	T4
e9	T4	T4	T4	T4	T4	T5	T4	T4	T5	T5	T5	...	T4	T4
e10	T4	T4	T4	T4	T4	T4	T5	T4	T4	T5	T5	...	T4	T4
e11	T4	T4	T4	T4	T4	T4	T4	T5	T4	T4	T5	...	T4	T4
e12	T4	T4	T4	T4	T4	T4	T4	T4	T5	T4	T4	...	T4	T4
e13	T4	T4	T4	T4	T4	T4	T4	T4	T4	T5	T4	...	T4	T4
e14	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	T5	...	T4	T4
e15	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	...	T4	T4
e16	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	...	T4	T4
e17	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	...	T4	T4
e18	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	...	T4	T4
e19	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	...	T4	T4
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
e381	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	...	T4	T5
e382	T5	T4	T4	T4	T4	T4	T4	T4	T4	T4	T4	...	T5	T4
e383	T4	T5	T4	T4	T4	T4	T4	T4	T4	T4	T4	...	T4	T5
e384	T5	T4	T5	T4	T4	T4	T4	T4	T4	T4	T4	...	T4	T4
e385	T4	T5	T4	T5	T4	T4	T4	T4	T4	T4	T4	...	T4	T4
e386	T4	T4	T5	T4	T5	T4	T4	T4	T4	T4	T4	...	T4	T4
e387	T4	T4	T4	T5	T4	T5	T4	T4	T4	T4	T4	...	T4	T4
e388	T4	T4	T4	T4	T5	T4	T5	T4	T4	T4	T4	...	T5	T4
e389	T4	T4	T4	T4	T4	T5	T4	T5	T4	T4	T4	...	T4	T5
e390	T5	T4	T4	T4	T4	T4	T5	T4	T5	T4	T4	...	T4	T4
e391	T4	T5	T4	T4	T4	T4	T4	T5	T4	T5	T4	...	T4	T4
e392	T4	T4	T5	T4	T4	T4	T4	T4	T5	T4	T5	...	T4	T4
e393	T4	T4	T4	T5	T4	T4	T4	T4	T4	T5	T4	...	T4	T4
e394	T4	T4	T4	T4	T5	T4	T4	T4	T4	T4	T5	...	T4	T4
e395	T4	T4	T4	T4	T4	T5	T4	T4	T4	T4	T4	...	T4	T4
e396	T4	T4	T4	T4	T4	T4	T5	T4	T4	T4	T4	...	T4	T4
e397	T4	T4	T4	T4	T4	T4	T4	T5	T4	T4	T4	...	T5	T4
e398	T4	T4	T4	T4	T4	T4	T4	T4	T5	T4	T4	...	T5	T5
e399	T5	T4	T4	T4	T4	T4	T4	T4	T5	T4	T5	...	T5	T5
e400	T5	T5	T4	T4	T4	T4	T4	T4	T4	T5	T5	...	T4	T5

Combine  $s_i$  with a binary system projective matrix we get a zero-one

$$matrix\ s_i = [T^5, T^5, T^5, T^5, T^5, \dots, \dots, \dots, T^5, T^5, T^5, T^5]$$

Let  $j_i = s + PLAN_i \quad 1 \leq i \leq 400$

Table (3,7) Now the table of  $j_i$



+	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	...	P399	P400
j1	T2	T2	T2	T5	T5	T5	T5	T5	T5	T5	...	T5	T5
j2	T5	T2	T2	T2	T5	T5	T5	T5	T5	T5	...	T2	T5
j3	T5	T5	T2	T2	T2	T5	T5	T5	T5	T5	...	T5	T2
j4	T2	T5	T5	T2	T2	T2	T5	T5	T5	T5	...	T5	T5
j5	T5	T2	T5	T5	T2	T2	T2	T5	T5	T5	...	T5	T5
j6	T5	T5	T2	T5	T5	T2	T2	T2	T5	T5	...	T5	T5
j7	T5	T5	T5	T2	T5	T5	T2	T2	T2	T5	...	T2	T5
j8	T5	T5	T5	T5	T2	T5	T2	T2	T2	T2	...	T5	T5
j9	T5	T5	T5	T5	T5	T2	T5	T5	T2	T2	...	T5	T5
j10	T5	T5	T5	T5	T5	T5	T2	T5	T5	T2	...	T5	T5
j11	T5	T5	T5	T5	T5	T5	T5	T2	T5	T5	...	T5	T5
j12	T5	T5	T5	T5	T5	T5	T5	T5	T2	T5	...	T5	T5
j13	T5	T5	T5	T5	T5	T5	T5	T5	T5	T2	...	T5	T5
j14	T5	T5	T5	T5	T5	T5	T5	T5	T5	T5	...	T5	T5
j15	T5	T5	T5	T5	T5	T5	T5	T5	T5	T5	...	T5	T5
j16	T5	T5	T5	T5	T5	T5	T5	T5	T5	T5	...	T2	T5
j17	T5	T5	T5	T5	T5	T5	T5	T5	T5	T5	...	T2	T5
j18	T5	T5	T5	T5	T5	T5	T5	T5	T5	T5	...	T2	T5
j19	T5	T5	T5	T5	T5	T5	T5	T5	T5	T5	...	T5	T5
...	...	...	...	...	...	...	...	...	...	...	...	...	...
j381	T5	T5	T5	T5	T5	T5	T5	T5	T5	T5	...	T5	T2
j382	T2	T5	T5	T5	T5	T5	T5	T5	T5	T5	...	T5	T5
j383	T5	T2	T5	T5	T5	T5	T5	T5	T5	T5	...	T2	T5
j384	T2	T5	T2	T5	T5	T5	T5	T5	T5	T5	...	T5	T2
j385	T5	T2	T5	T2	T5	T5	T5	T5	T5	T5	...	T5	T5
j386	T5	T5	T2	T5	T2	T5	T5	T5	T5	T5	...	T5	T5
j387	T5	T5	T5	T2	T5	T2	T5	T5	T5	T5	...	T5	T5
j388	T5	T5	T5	T5	T2	T5	T2	T5	T5	T5	...	T5	T5
j389	T5	T5	T5	T5	T5	T2	T5	T2	T5	T5	...	T5	T5
j390	T2	T5	T5	T5	T5	T5	T2	T5	T2	T5	...	T5	T5
j391	T5	T2	T5	T5	T5	T5	T5	T2	T5	T2	...	T5	T5
j392	T5	T5	T2	T5	T5	T5	T5	T5	T2	T5	...	T5	T5
j393	T5	T5	T5	T2	T5	T5	T5	T5	T2	T5	...	T5	T5
j394	T5	T5	T5	T5	T2	T5	T5	T5	T5	T5	...	T5	T5
j395	T5	T5	T5	T5	T5	T2	T5	T5	T5	T5	...	T5	T5
j396	T5	T5	T5	T5	T5	T5	T2	T5	T5	T5	...	T5	T5
j397	T5	T5	T5	T5	T5	T5	T5	T2	T5	T5	...	T2	T5
j398	T5	T5	T5	T5	T5	T5	T5	T5	T2	T5	...	T2	T2
j399	T2	T5	T5	T5	T5	T5	T5	T5	T5	T2	...	T2	T2
j400	T2	T2	T5	T5	T5	T5	T5	T5	T5	T5	...	T5	T2

Find the shortest distance between two different code word in the matrices above, so that the shortest distance is 57 and largest distance is 400 .  $m_i, v_i, h_i, k_i, o_i, w_i, s_i, l_i, a_i, b_i, c_i, d_i, e_i, j_i$

Table 4

$d(m_i, PLAN_i) = 57$	$d(e_i, m_i) = 400$	$d(o_i, k_i) = 400$	$d(j_i, w_i) = 400$
$d(v_i, PLAN_i) = 343$	$d(j_i, m_i) = 400$	$d(w_i, k_i) = 400$	$d(a_i, s_i) = 400$
$d(h_i, PLAN_i) = 400$	$d(h_i, v_i) = 400$	$d(s_i, k_i) = 400$	$d(b_i, s_i) = 400$
$d(k_i, PLAN_i) = 400$	$d(k_i, v_i) = 400$	$d(a_i, k_i) = 400$	$d(c_i, s_i) = 400$
$d(o_i, PLAN_i) = 400$	$d(o_i, v_i) = 400$	$d(b_i, k_i) = 400$	$d(d_i, s_i) = 400$
$d(w_i, PLAN_i) = 400$	$d(w_i, v_i) = 400$	$d(c_i, k_i) = 57$	$d(e_i, s_i) = 343$
$d(s_i, PLAN_i) = 400$	$d(s_i, v_i) = 400$	$d(d_i, k_i) = 400$	$d(j_i, s_i) = 57$
$d(a_i, PLAN_i) = 400$	$d(a_i, v_i) = 343$	$d(e_i, k_i) = 400$	$d(b_i, a_i) = 400$
$d(b_i, PLAN_i) = 400$	$d(b_i, v_i) = 400$	$d(j_i, k_i) = 400$	$d(c_i, a_i) = 400$
$d(c_i, PLAN_i) = 400$	$d(c_i, v_i) = 400$	$d(w_i, o_i) = 400$	$d(d_i, a_i) = 400$
$d(d_i, PLAN_i) = 400$	$d(d_i, v_i) = 400$	$d(s_i, o_i) = 400$	$d(e_i, a_i) = 400$
$d(e_i, PLAN_i) = 400$	$d(e_i, v_i) = 400$	$d(a_i, o_i) = 400$	$d(j_i, a_i) = 400$
$d(j_i, PLAN_i) = 400$	$d(j_i, v_i) = 400$	$d(b_i, o_i) = 343$	$d(c_i, b_i) = 400$
$d(v_i, m_i) = 400$	$d(k_i, h_i) = 400$	$d(c_i, o_i) = 400$	$d(d_i, b_i) = 400$
$d(h_i, m_i) = 400$	$d(o_i, h_i) = 400$	$d(d_i, o_i) = 57$	$d(e_i, b_i) = 400$
$d(k_i, m_i) = 400$	$d(w_i, h_i) = 400$	$d(e_i, o_i) = 400$	$d(j_i, b_i) = 400$





$d(o_i, m_i) = 400$	$d(s_i, h_i) = 400$	$d(j_i, o_i) = 400$	$d(d_i, c_i) = 400$
$d(w_i, m_i) = 400$	$d(a_i, h_i) = 400$	$d(s_i, w_i) = 400$	$d(e_i, c_i) = 400$
$d(s_i, m_i) = 400$	$d(b_i, h_i) = 57$	$d(a_i, w_i) = 343$	$d(j_i, c_i) = 400$
$d(a_i, m_i) = 400$	$d(c_i, h_i) = 343$	$d(b_i, w_i) = 400$	$d(e_i, d_i) = 400$
$d(b_i, m_i) = 400$	$d(d_i, h_i) = 400$	$d(c_i, w_i) = 400$	$d(j_i, d_i) = 400$
$d(c_i, m_i) = 400$	$d(e_i, h_i) = 400$	$d(d_i, w_i) = 400$	$d(j_i, e_i) = 400$
$d(d_i, m_i) = 57$	$d(j_i, h_i) = 400$	$d(e_i, w_i) = 57$	

If we substitute the values of  $n = 400$ ,  $d = 57$ ,  $e = 28$  in inequality of Theorem 3.1 we get  $M = 7^{396}$   
Hence C is a  $(400, 7^{396}, 57)$  - code.

$$M \left\{ \binom{n}{0} + \binom{n}{1}(q-1) + \dots + \binom{n}{e}(q-1)^e \right\} \leq q^n$$

As  $n = q^3 + q^2 + q + 1$

And  $M = q^{q^3+q^2+q+1-k}$

Then

$$M \left\{ \binom{n}{0} + \binom{n}{1}(q-1) + \dots + \binom{n}{e}(q-1)^e \right\} \leq q^n$$

Become

$$q^{q^3+q^2+q+1-k} \left\{ \binom{n}{0} + \binom{n}{1}(q-1) + \dots + \binom{n}{e}(q-1)^e \right\} \leq q^{q^3+q^2+q+1}$$

$$q^{q^3+q^2+q+1} q^{-k} \left\{ \binom{n}{0} + \binom{n}{1}(q-1) + \dots + \binom{n}{e}(q-1)^e \right\} \leq q^{q^3+q^2+q+1}$$

$$\left\{ \binom{n}{0} + \binom{n}{1}(q-1) + \dots + \binom{n}{e}(q-1)^e \right\} \leq q^k$$

$$\begin{aligned} & \left\{ \binom{400}{0} + \binom{400}{1}(7-1) + \binom{400}{2}(7-1)^2 + \binom{400}{3}(7-1)^3 + \binom{400}{4}(7-1)^4 + \right. \\ & \binom{400}{5}(7-1)^5 + \binom{400}{6}(7-1)^6 + \binom{400}{7}(7-1)^7 + \binom{400}{8}(7-1)^8 + \binom{400}{9}(7-1)^9 + \\ & \binom{400}{10}(7-1)^{10} + \binom{400}{11}(7-1)^{11} + \binom{400}{12}(7-1)^{12} + \binom{400}{13}(7-1)^{13} + \binom{400}{14}(7-1)^{14} + \\ & \binom{400}{15}(7-1)^{15} + \binom{400}{16}(7-1)^{16} + \binom{400}{17}(7-1)^{17} + \binom{400}{18}(7-1)^{18} + \binom{400}{19}(7-1)^{19} + \\ & \binom{400}{20}(7-1)^{20} + \binom{400}{21}(7-1)^{21} + \binom{400}{22}(7-1)^{22} + \binom{400}{23}(7-1)^{23} + \binom{400}{24}(7-1)^{24} + \\ & \left. \binom{400}{25}(7-1)^{25} + \binom{400}{26}(7-1)^{26} + \binom{400}{27}(7-1)^{27} + \binom{400}{28}(7-1)^{28} \right\} \leq 7^4 \text{ but that is} \\ & \text{not true} \end{aligned}$$

Then is not perfect

#### 4-Codes by Groups Action on The PG(3,7)

Recall that, if  $d = n - k + 1$ , then  $[n, k, d]_q$  - code is called (MDS) maximum distance separable, and if  $d = n - k$ , then the code is called almost maximum distance separable (AMDS). Additionally, if C is an  $[n, k, n - k + 1]_q$  - code then  $C^\perp$  is an  $[n, n - k, k + 1]_q$  - code



Dual code  $C^\perp$  of an  $[n, k, d]_7$  – codes  $C$  over  $F_7$

$C = [400.4.57]_7$  – codes

$C^\perp = [400.396.5]_7$  – codes is (MDS)

The main Coding Theory problem is to find "good" codes, those which

maximize both  $R$  and  $\delta$ , where  $R = \frac{k}{n}$  is the information rate and the relative

distance is  $\delta = \frac{d}{n}$

then in  $[400.4.57]_7$  – codes  $R=0.01$  and  $\delta= 0.1425$

and in  $[400.396.5]_7$  – codes  $R=0.99$  and  $\delta= 0.0125$

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